

CHAPITRE II

DEFORMATIONS

CHAPITRE II : LES DEFORMATIONS

I. Introduction

II. Gradient d'une transformation

$$\begin{aligned} & \mathbf{F} : \mathbf{x} \mapsto \mathbf{x}' \\ & \begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \end{aligned}$$

III. Tenseur des dilatations

$$\mathbf{D} = \frac{1}{3} \text{tr}(\mathbf{D}) \mathbf{1} + \mathbf{D}' + \mathbf{D}''$$

IV. Déformations

$$\begin{aligned} & \mathbf{D} = \frac{1}{3} \text{tr}(\mathbf{D}) \mathbf{1} + \mathbf{D}' + \mathbf{D}'' \\ & \text{tr}(\mathbf{D}) = \lambda_1 + \lambda_2 + \lambda_3 \end{aligned}$$

$$\begin{aligned} & \mathbf{D}' = \frac{1}{2} (\mathbf{D} + \mathbf{D}^T) \\ & \mathbf{D}'' = \frac{1}{2} (\mathbf{D} - \mathbf{D}^T) \end{aligned}$$

V. Formulation en fonction des déplacements

$$\begin{aligned} & \mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \\ & \mathbf{D} = \frac{1}{2} (\nabla \mathbf{u} + \nabla \mathbf{u}^T) \end{aligned}$$

VI. Déformation en petite transformation

$$\begin{aligned} & \mathbf{D} = \frac{1}{2} (\nabla \mathbf{u} + \nabla \mathbf{u}^T) \\ & \mathbf{D}'' = \frac{1}{2} (\nabla \mathbf{u} - \nabla \mathbf{u}^T) \end{aligned}$$

VII. Vitesses de déformation

$$\dot{\mathbf{D}}$$

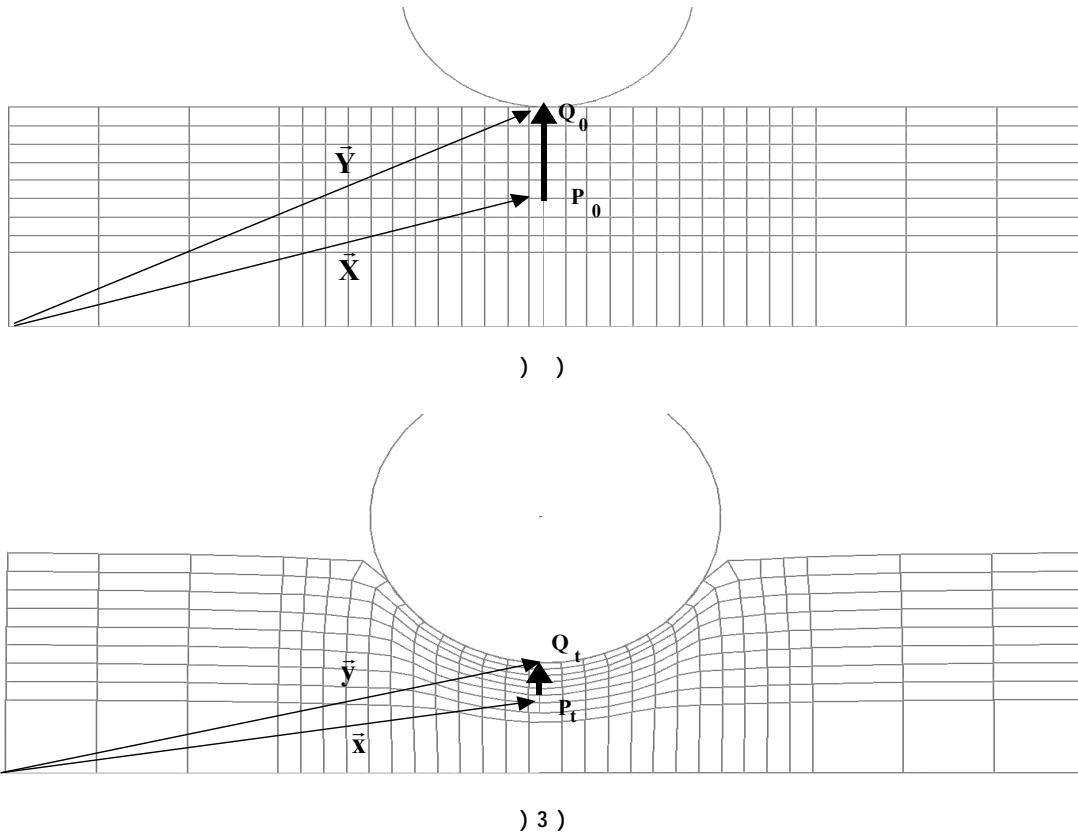


Figure 1 : Réseau de lignes dessinées sur un solide (a) avant indentation et (b) après indentation par un cylindre.

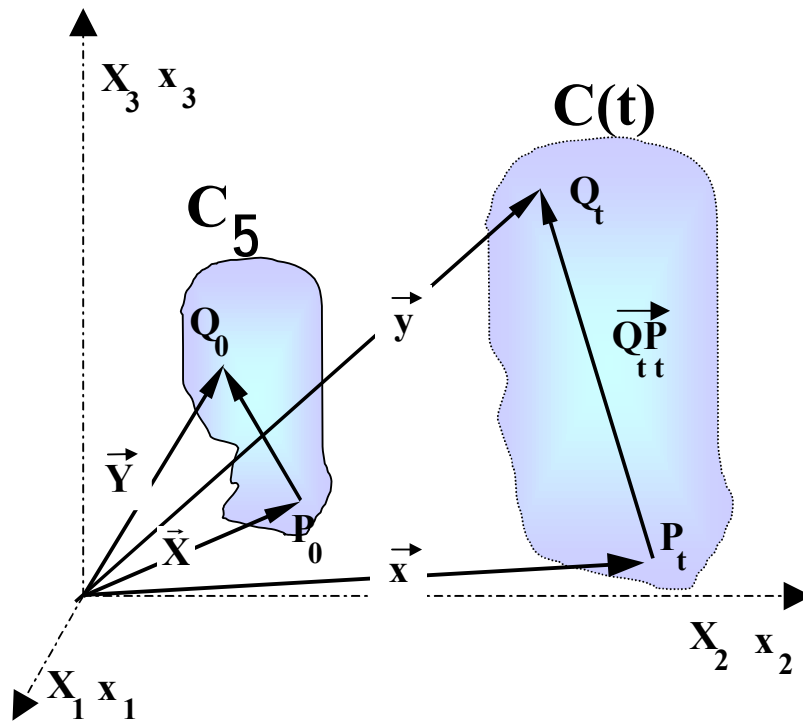


Figure 2 : Configuration initiale et actuelle d'un solide déformé.

I. INTRODUCTION

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II. GRADIENT D'UNE TRANSFORMATION

II.1. Tenseur gradient d'une transformation

\vec{e}_i \vec{e}_j \vec{e}_k \vec{e}_l \vec{e}_m ;
 δ_{ij} δ_{jk} δ_{kl} δ_{lm} δ_{mn} ;
 δ_{ij} δ_{jk} δ_{kl} δ_{lm} δ_{mn} ;
 δ_{ij} δ_{jk} δ_{kl} δ_{lm} δ_{mn} ;
 δ_{ij} δ_{jk} δ_{kl} δ_{lm} δ_{mn} ;

$$\vec{e}_i = \vec{e}_j \delta_{ij} \quad ; \quad \vec{e}_j = \vec{e}_i \delta_{ji}$$

\vec{e}_i \vec{e}_j \vec{e}_k \vec{e}_l \vec{e}_m ;
 δ_{ij} δ_{jk} δ_{kl} δ_{lm} δ_{mn} ;
 δ_{ij} δ_{jk} δ_{kl} δ_{lm} δ_{mn} ;
 δ_{ij} δ_{jk} δ_{kl} δ_{lm} δ_{mn} ;
 δ_{ij} δ_{jk} δ_{kl} δ_{lm} δ_{mn} ;

$$\vec{e}_i = \vec{e}_j \delta_{ij} = (\vec{e}_k \delta_{kj}) \delta_{ij} = \vec{e}_k \delta_{ki} \delta_{ij} = \vec{e}_k \delta_{kj} \delta_{ij}$$

\vec{e}_i \vec{e}_j \vec{e}_k \vec{e}_l \vec{e}_m ;
 δ_{ij} δ_{jk} δ_{kl} δ_{lm} δ_{mn} ;
 δ_{ij} δ_{jk} δ_{kl} δ_{lm} δ_{mn} ;
 δ_{ij} δ_{jk} δ_{kl} δ_{lm} δ_{mn} ;
 δ_{ij} δ_{jk} δ_{kl} δ_{lm} δ_{mn} ;

$$\left(\frac{\partial \Phi}{\partial x_i} \right) = \left(\frac{\partial \Phi}{\partial x_j} \right) \delta_{ij} = \left(\frac{\partial \Phi}{\partial x_k} \right) \delta_{kj} \delta_{ij} = \left(\frac{\partial \Phi}{\partial x_k} \right) \delta_{ki} \delta_{ij} = \left(\frac{\partial \Phi}{\partial x_k} \right) \delta_{kj} \delta_{ij}$$

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$$\begin{aligned} \bar{y} - \bar{x} &= F(\bar{X}) (\bar{Y} - \bar{X}) + \bar{\alpha} (\bar{Y} - \bar{X}) \|\bar{Y} - \bar{X}\| \\ P_t Q_t &= F(P_5 Q_5) P_5 Q_5 + \bar{\alpha} (P_5 Q_5) \|P_5 Q_5\| \end{aligned}$$

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$\underline{E} \bar{X} = \underline{D} \bar{\Phi} \bar{X}$, \$ <5 , \$ E 4 E

$$\begin{aligned} \underline{F} \bar{X} &= \overline{\text{grad}} \bar{\Phi} \bar{X} = \overline{\text{grad}} \bar{x} \bar{X} \\ F_{ij} &= \frac{\partial x_i}{\partial X_j} \end{aligned}$$

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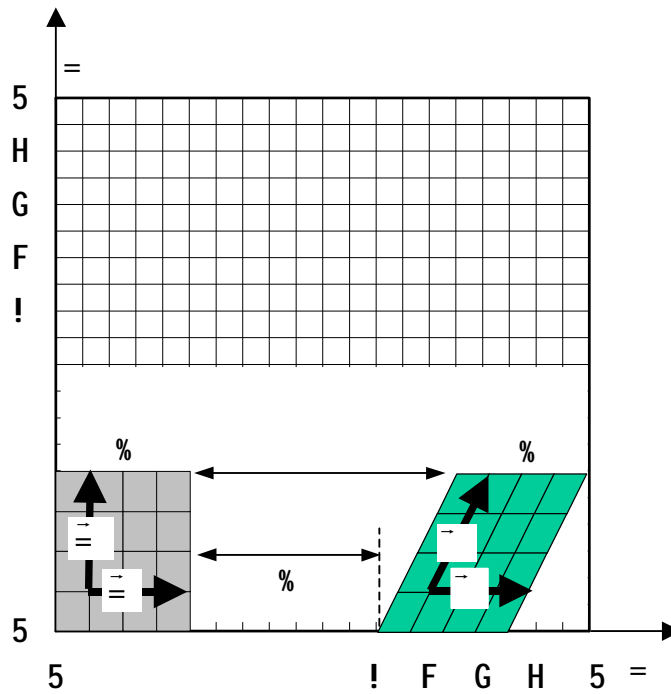


Figure 3 : Exemple de transformation particulière : glissement simple avec translation

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$\underline{\underline{E}}$

$\&5\%$

transformation homogène :

$$\bar{x} = \bar{\Phi} \bar{X} = \underline{\underline{F}} \bar{X} + \bar{B}$$

$$x_i = \Phi_i X_j = F_{ij} X_j + B_i$$

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, 7 8 # \$ \$ # $\overrightarrow{\< = \underline{\underline{E}}(\) \>}$
 , \$ $\bar{\alpha}$

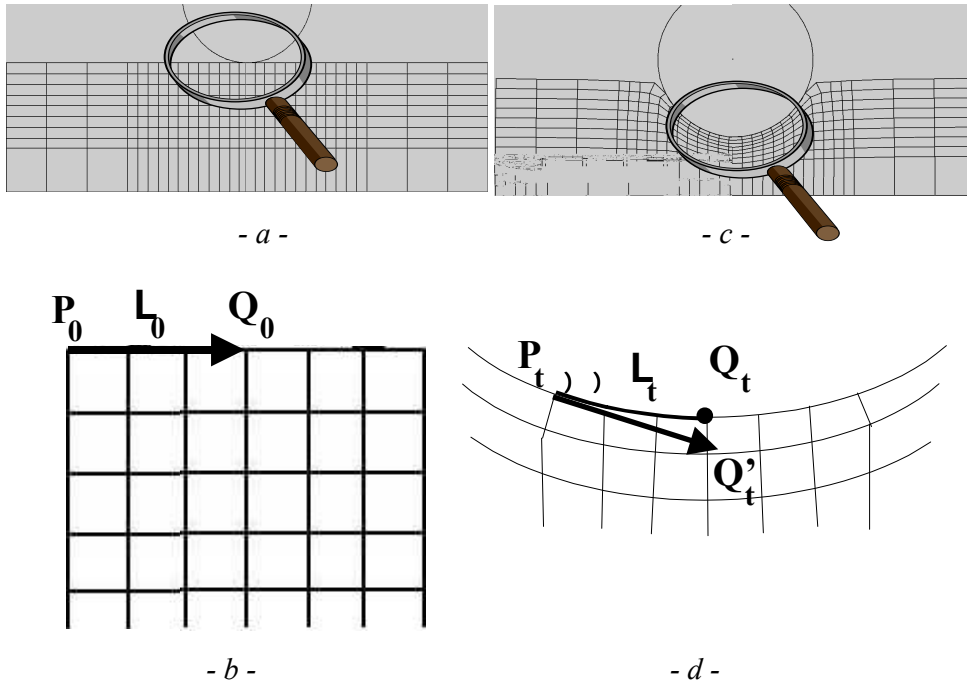


Figure 4: Indentation d'un solide par un cylindre infini en configuration non déformée prise comme référence (a et b) et configuration déformée (c et d).

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$L_0 \mathbf{B}_{5 < 5} \quad \$$

$\$ \quad C78 \quad O$

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$$= \underline{\underline{E}}(\overrightarrow{5\&5\%}) \overrightarrow{\&5 < 5} + \bar{\alpha}(\overrightarrow{\&5 < 5\%}) \|\overrightarrow{\&5 < 5}\|$$

$$\overrightarrow{5} ; \quad \$ \quad 4 \overrightarrow{\& <} = \overrightarrow{\& < J} + \bar{\alpha}(\overrightarrow{\&5 < 5\%}) \|\overrightarrow{\&5 < 5}\|$$

$$\bar{5} \quad \|\overrightarrow{\&5 < 5}\| \quad " \quad \bar{5} * \quad \% \quad K \quad < J$$

$\&5 \quad 1$

$\# \quad \$ \quad " \quad \$ \quad \overrightarrow{\& <}$

$" \quad \$ \quad \overrightarrow{\& < J} ;$

$\overrightarrow{\&5 < 5} * \# \quad \% \quad " \quad \$ \quad \overrightarrow{\& < J}$

$\$$

$" \quad \$ \quad " \quad \$$

$L_5 \mathbf{B}_{5 < 5}$

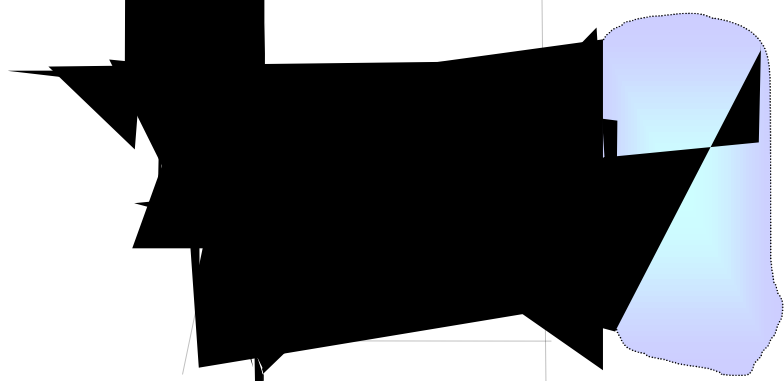
$\# \quad \$$

transporté du vecteur $\overrightarrow{P_5 Q_5}$

$$\overrightarrow{Q_t} = \underline{\underline{F}}(\overrightarrow{5P_5\%}) \overrightarrow{P_5 Q_5}$$

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$$\overrightarrow{Q_t} = \overrightarrow{P_t Q_t} + \bar{\alpha}(\overrightarrow{P_5 Q_5 \%}) \|\overrightarrow{P_5 Q_5}\|$$



$$\begin{aligned} & \text{, \#} \quad 7 \ 8 \quad \$ \\ & \text{" \$} \quad \overrightarrow{\< J} = \underline{\underline{E}} \left(\overrightarrow{5 \&5 \%} \right) \overrightarrow{\&5 < 5} \quad 3 \quad \text{" \$} \\ & \overrightarrow{\<} \quad <5 \quad \$ \quad \&5 \quad \% \quad \$ \quad \&5 \quad <5 \\ & \quad \$ \ \% \quad 4 \quad \overrightarrow{\&5 < 5} = \overrightarrow{\&5 \%} \quad \overrightarrow{\< J} = \overrightarrow{\&5 \#} \quad 7E \quad 8 \\ & \overrightarrow{\<} = \overrightarrow{\< -}; \quad \& = \overrightarrow{\< -} = \Delta^- \quad \overrightarrow{\&5 < 5} = \overrightarrow{\< 5 -}; \quad \overrightarrow{\&5} = \overrightarrow{\< -} = \overrightarrow{\&5 \%} \# \quad 7 \ 8 \quad \text{"} \quad 4 \end{aligned}$$

$$\Delta^- = \overrightarrow{\< -} + \overrightarrow{\&5} \left(\overrightarrow{\&5} \right) \overrightarrow{\&5}$$

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$$\begin{aligned} O \quad \text{" \$} \quad \overrightarrow{\<} \quad \text{" \$} \quad \overrightarrow{\< J} \quad \text{"} \quad \$ * \quad \$ \quad \# \$ \$ \quad \Delta^- \\ \$ \quad \overrightarrow{\&5} = \% 8 \quad \text{"} \quad \$ \quad - \end{aligned}$$

$$O \quad \text{"} \quad \text{"} \quad 4$$

$$\delta ? = \frac{\partial =}{\partial = ?} = \frac{\partial =}{\partial} \frac{\partial}{\partial = ?}$$

$$A \delta ? \quad (\ 3 \quad L \quad \$ M \ 7 \delta ? = \overrightarrow{\&5} \delta ? = 5 \neq ? 8 \ , \ # \ \text{"} \quad \underline{\underline{E}}$$

$$\$ \quad 4$$

$$\delta ? = \left(E^- \right) ? = \frac{\partial =}{\partial ?}$$

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II.2. Transport convectif

$$\begin{aligned} & \$ \quad \$ \ C_5 \% \quad \$ \quad \$ \ 3 \ L_5 \quad \&5 \ - \\ & \text{"} \quad \&5 \quad \$ \quad \$ \quad \$ \quad \text{"} \quad \$ \quad \overrightarrow{\&5} \quad \overrightarrow{\&5} \quad 6 \\ & \text{"} \quad \$ \quad 1 \quad \$ \quad 6 \quad \$ \\ & * \quad \text{"} \quad \$ \quad \overrightarrow{\&5} \quad \overrightarrow{\&5} \quad \&5 \ 1 \quad \text{"} \\ & \quad +5 \quad \$ \quad * \quad \text{"} \quad \$ \quad \overrightarrow{\&5} \% \end{aligned}$$

\vec{J} \vec{J} $\$$ $\$$ $C78\%$ L $\# \$ L_5$ $\$$
 $" \$$ $" \$$ $-\%$ $" \$$ $\vec{=}$ $,$ 6 $\$$ 6_5
 $\$$ $" \$$ $\$$ $" \$$ $-$ $-J\%$ $\vec{=}$ $\vec{=}$

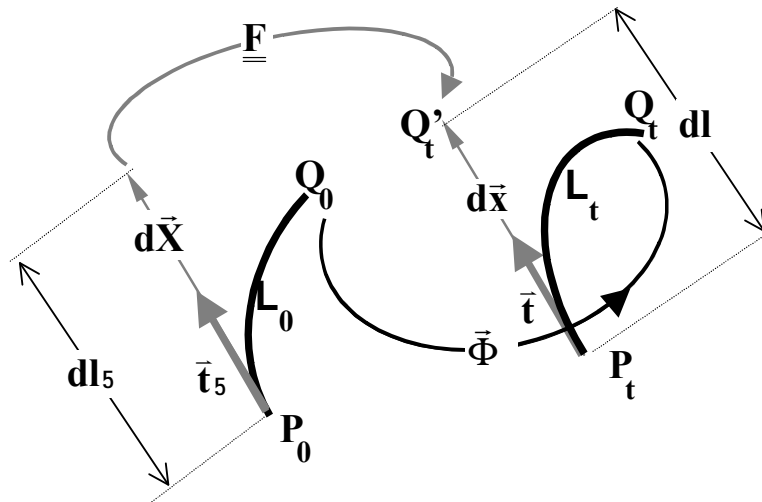


Figure 6 : transport d'un vecteur élémentaire

0 $\$$ $\$$ $3 L_5$ $\&_5$ $<_5 7E$ $!8$
 $\$$ $\$$ $\$$ $\$$ $\% \# \$$ $\$$ $3 L_5$
 $\# \$$ $\$$ $3 L$ $\& < 0$ $\$$ $\$$ 3 $\$$
 $" \$$ $\vec{=}$ $-$ $" \$$ $-_5$ $-$ $\vec{=}$ $5 -_5$
 $- = - 0$ $- = \vec{E} - \%$ $\$$ 4

$$\vec{=} = \vec{E} - 5 \ 5$$

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$6 \ 6_5$ $\$$ $\$$ $*$ $" \$$
 $\vec{=}$ $\vec{=}$ $6 -_5$ $\#$ $6_5 -_5$ $*$ 6_5 4
 $-_5 -_5 = \vec{=} \wedge \vec{=}$
 $\$$ $6 \$$ $*$ $" \$$ $- = \vec{E} -$ $-J = \vec{E} - \vec{J}$
 $\$$ $6_5 \&$ 3 6 65%

$\vec{\epsilon}_M = \epsilon_M \vec{e}_M$
 $\vec{\epsilon}_M = \epsilon_M \vec{e}_M$
 $\vec{\epsilon}_M = -\epsilon_M \vec{e}_M$

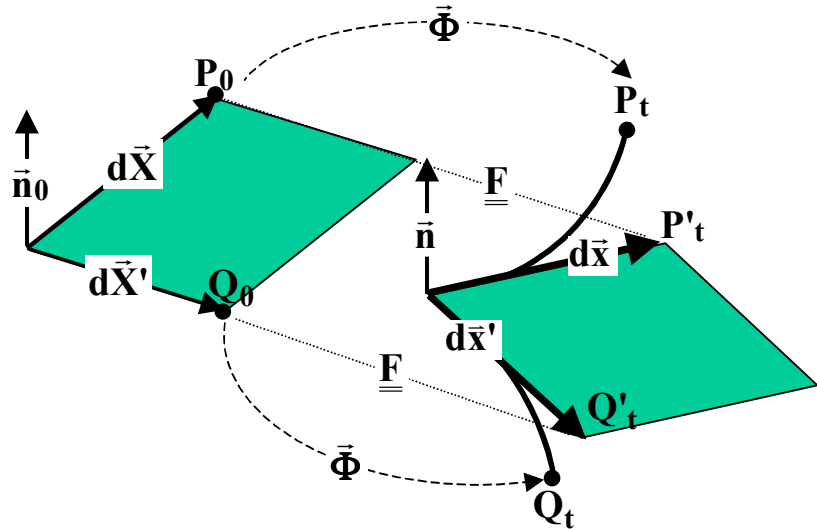


Figure 7: Transport d'un élément de surface

$$\vec{E} = \vec{E}_t \quad J_t = E_t \quad \vec{J} \quad \vec{E} = \vec{E}_t \quad J_M = E_M \quad = J$$

$$-\epsilon_M (E_t =) (E_M = J)$$

$\& \quad K \quad \# \quad \underline{E} \quad \text{\$N}$
 $\# \quad E \quad \# \quad \text{\$}$

$$E \quad - = \epsilon_M E \quad E_t \quad E_M \quad = \quad = J$$

$$; \quad \epsilon_M E \quad E_t \quad E_M = \epsilon \quad \underline{E} = \epsilon \quad 0 \quad 7- \quad 8\% \quad \text{\$}$$

$$E \quad - = \epsilon \quad 0 \quad = \quad = J$$

$$O \quad \# \quad - \quad \text{\$} \quad -5 \quad -5 \quad \% \quad \vec{e} \wedge \vec{e} = \vec{e} = -5 \quad -5$$

$$\#\text{\$} \quad \text{\$} \quad 5 \quad -5 = \epsilon \quad = \quad = J \quad O \quad \#\text{\$} \quad 4$$

$$E \quad - = 0 \quad 5 \quad -5$$

$$E = E = \left[\underline{E} \right]$$

, \$ \$ # \$

$$\underline{E} - = 0 - 5 - 5\%$$

$$\underline{E} - = 0 - 5 - 5\% = \underline{E}$$

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 " & 5 \$ \$ \$ " \$ = = 6
 = =J & 5 + 5 = % =J 07 8% , # \$, 5 \$ " \$
 " \$ - % " \$ = , 6 \$ 65 \$
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III. TENSEUR DES DILATATIONS

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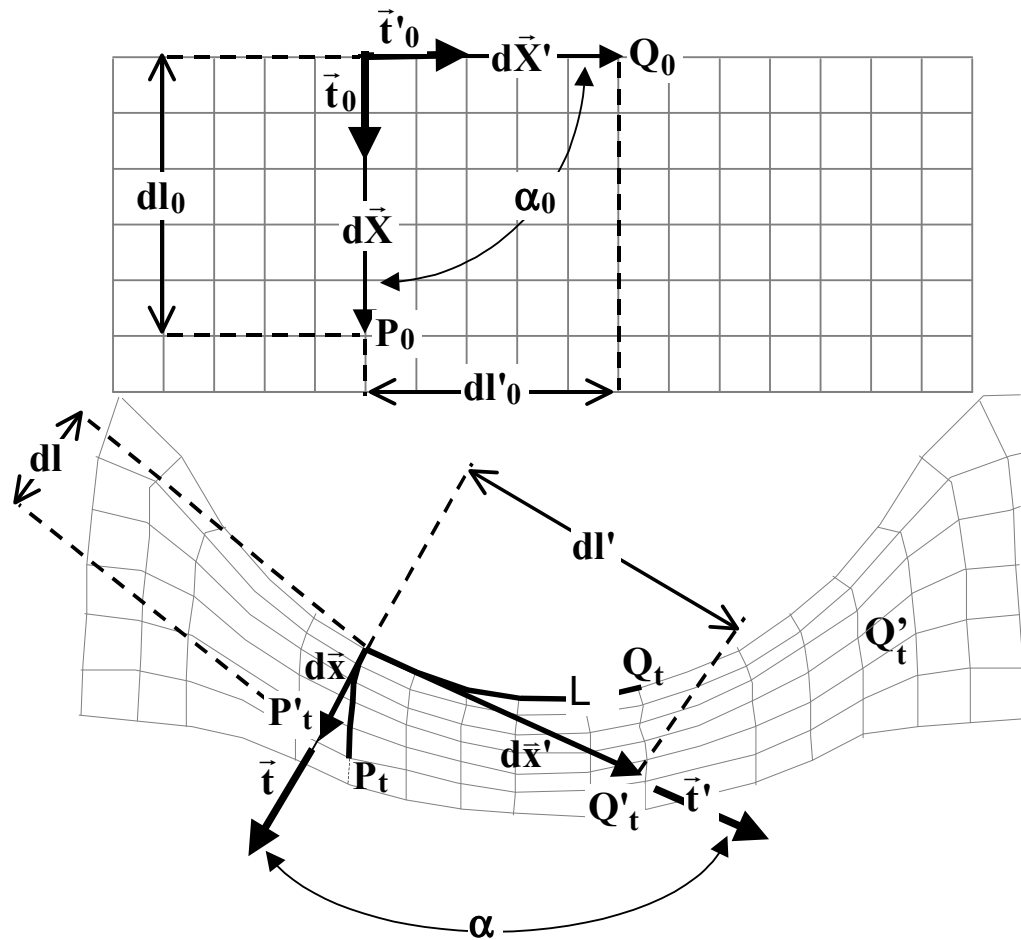


Figure 9: Indentation d'un solide par un cylindre infinie. Configuration de référence et configuration déformée.

III.1. Tenseur des dilatations

$$\underline{O} = \underline{E} \underline{E} = \underline{E} \underline{E} = \underline{O}$$

$$\underline{O} = \underline{E} \underline{E} = \underline{E} \underline{E} = \underline{O}$$

$$\underline{O} = \underline{E} \underline{E} = \underline{E} \underline{E} = \underline{O}$$

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$$\underline{O} = \underline{E} \underline{E} = \underline{E} \underline{E} = \underline{O}$$

$$\underline{O} = \underline{E} \underline{E} = \underline{E} \underline{E} = \underline{O}$$

$$\underline{O} = \underline{E} \underline{E} = \underline{E} \underline{E} = \underline{O}$$

$$\underline{O} = \underline{E} \underline{E} = \underline{E} \underline{E} = \underline{O}$$

$$\frac{\partial}{\partial \underline{\underline{E}}} = \underline{\underline{E}} \underline{\underline{E}}$$

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0 \$ 0' * \$ % \$ \$

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$$\underline{\underline{\$}} = \left(\frac{\partial}{\partial \underline{\underline{E}}} \right)^{-1} = \left[\underline{\underline{E}} \underline{\underline{E}} \right]^{-1} = \left[\underline{\underline{E}}^{-1} \right] \underline{\underline{E}}^{-1} = \underline{\underline{E}}^{-1} \underline{\underline{E}}^{-1} \underline{\underline{\$}}$$

$$\underline{\underline{\$}} = \underline{\underline{E}}^{-1} \underline{\underline{E}}^{-1} \underline{\underline{\$}}$$

$$\underline{\underline{\$}} = \underline{\underline{E}}^{-1} \underline{\underline{E}}^{-1} \underline{\underline{\$}}$$

$$\lambda(\vec{v}) = \lambda(\vec{v}_5) \begin{cases} = \sqrt{\frac{-5}{5}} \vec{v}_5 \\ = \frac{1}{\sqrt{5}} \vec{v}_5 \\ = \left\| \frac{\vec{v}_5}{\sqrt{5}} \right\| \end{cases}$$

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 # \$ = &5 λ \$ = %
 &5 # O# \$ %\$ O ;
 \$ -5 : λ 7 -5 8 * \$ \$
 \$ \$ \$ 6 -5 \$ *
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III.3. Glissement de deux directions orthogonales

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 " \$ = =J α5 \$ \$
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$$\alpha = \frac{\vec{v}_i \cdot \vec{v}_j}{\|\vec{v}_i\| \|\vec{v}_j\|} = \frac{(\vec{e}_i) \cdot (\vec{e}_j)}{\sqrt{(\vec{e}_i) \cdot (\vec{e}_i)} \sqrt{(\vec{e}_j) \cdot (\vec{e}_j)}}$$

$$\alpha = \frac{\vec{v}_i \cdot \vec{v}_j}{\|\vec{v}_i\| \|\vec{v}_j\|} = \frac{(\vec{e}_i \ \vec{e}_j) \cdot \vec{v}_j}{\sqrt{(\vec{e}_i \ \vec{e}_i)} \sqrt{(\vec{e}_j \ \vec{e}_j)}}$$

$$\alpha = \frac{\vec{v}_i \cdot \vec{v}_j}{\|\vec{v}_i\| \|\vec{v}_j\|} = \frac{(\vec{v}_i) \cdot (\vec{v}_j)}{\sqrt{[(\vec{v}_i) \cdot (\vec{v}_i)]} \sqrt{[(\vec{v}_j) \cdot (\vec{v}_j)]}}$$

$$\alpha = \frac{-\frac{1}{5} \pm \sqrt{\frac{1}{25} - \frac{1}{5}}}{\frac{1}{5}} = \frac{-1 \pm \sqrt{1 - 5}}{1}$$

$$\alpha = \frac{-\frac{1}{5} \pm \sqrt{\frac{1}{25} - \frac{1}{5}}}{\frac{1}{5}} = \frac{-1 \pm \sqrt{1 - 5}}{1}$$

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IV. DEFORMATIONS
IV.1. Tenseurs des déformations

$$\begin{aligned} \underline{\underline{P}} &= \underline{\underline{J}}^{-1} \underline{\underline{P}} \underline{\underline{J}} = \underline{\underline{O}} \underline{\underline{J}} \underline{\underline{P}} \underline{\underline{J}}^{-1} \\ &= \underline{\underline{J}}^{-1} \underline{\underline{P}} \underline{\underline{J}} = \underline{\underline{O}} \underline{\underline{J}} \underline{\underline{P}} \underline{\underline{J}}^{-1} \\ &= \underline{\underline{J}}^{-1} \underline{\underline{P}} \underline{\underline{J}} = \underline{\underline{O}} \underline{\underline{J}} \underline{\underline{P}} \underline{\underline{J}}^{-1} \end{aligned}$$

$$\underline{\underline{P}} = -\underline{\underline{O}} \underline{\underline{J}} \underline{\underline{P}} \underline{\underline{J}}^{-1}$$

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$$\underline{\underline{O}} \underline{\underline{O}} \underline{\underline{O}} = \underline{\underline{E}} \underline{\underline{E}} \underline{\underline{E}} \%$$

E 4

$$\underline{\underline{P}} = -\underline{\underline{E}} \underline{\underline{E}} \underline{\underline{E}} \underline{\underline{P}}$$

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$$\underline{\underline{O}} \underline{\underline{O}} \underline{\underline{O}} = \underline{\underline{E}} \underline{\underline{O}} \underline{\underline{O}} \%$$

$$\underline{\underline{O}} \underline{\underline{O}} \underline{\underline{O}} = \underline{\underline{E}} \underline{\underline{O}} \underline{\underline{O}} \%$$

$$\begin{aligned} \underline{\underline{P}} &= \underline{\underline{J}}^{-1} \underline{\underline{P}} \underline{\underline{J}} = \underline{\underline{J}}^{-1} \underline{\underline{P}} \underline{\underline{J}} \\ &= \underline{\underline{J}}^{-1} \underline{\underline{P}} \underline{\underline{J}} = \underline{\underline{J}}^{-1} \underline{\underline{P}} \underline{\underline{J}} \\ &= \underline{\underline{J}}^{-1} \underline{\underline{P}} \underline{\underline{J}} = \underline{\underline{J}}^{-1} \underline{\underline{P}} \underline{\underline{J}} \end{aligned}$$

$$\begin{aligned}
 - \quad -J_1 &= \underline{\underline{-J}} = - \quad -J_1 (-) \quad \underline{\underline{-}} \quad - \\
 &= - \quad -J_1 - \underline{\underline{\$}} \quad -J \\
 - \quad -J_1 &= \underline{\underline{-J}} = - \quad \left[\underline{\underline{-\$}} \right] \quad -J
 \end{aligned}$$

$$\begin{aligned}
 & \quad \quad \quad J \quad - - \\
 = & = - \left[\underline{\underline{-\$}} \right] \\
 - \quad -J_1 &= \underline{\underline{-J}} = \underline{\underline{-J}} = - \quad - \quad -J
 \end{aligned}$$

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$$\begin{aligned}
 & \quad \quad \quad \$ \quad \underline{\underline{\$}}\% \quad \quad \quad \# \quad)- \quad = \quad \quad \quad \# \\
 3 & \quad \quad \quad \# \quad \quad \quad " \quad \$ \quad \quad \quad * \quad \$ \quad \quad \quad \$ \quad , \quad \$ \\
 & = \quad \quad \quad 3 \quad . \quad 4 \\
 & \quad \quad \quad J_1 - = \underline{\underline{-J}} = \quad \quad \quad J \\
 & \quad \quad \quad J_1 - = \underline{\underline{-J}} = \quad \quad \quad J \\
 & \quad \quad \quad ? \quad J_1 - = \underline{\underline{-J}} = \quad \quad \quad J \\
 & - \quad -J_1 - \underline{\underline{-}} = \underline{\underline{-J}} = \underline{\underline{-}} = \underline{\underline{-}}
 \end{aligned}$$

$$\begin{aligned}
 & \quad \quad \quad \& \quad 3 \quad \quad \quad \cdot \quad), \quad = \\
 & \quad \quad \quad \# \quad)- \quad = \% \quad \quad \quad \$ \\
 \$ & \quad \quad \quad \$ \quad \quad \quad \$ \quad \quad \quad \$ \quad \quad \quad \$ \quad \quad \quad \$ \\
 4 & \\
 & - \quad -J_1 - \underline{\underline{-}} = \underline{\underline{-J}} = \underline{\underline{-}} = \underline{\underline{-J}} = - \quad = \quad -J \\
 & \quad \quad \quad 3 \quad \quad \quad \$ \quad \quad \quad \$ \quad \quad \quad \underline{\underline{-}} \quad \underline{\underline{-}} \quad 0 \quad \$ \quad \$ \\
 * & \quad \quad \quad \underline{\underline{-}} \quad \underline{\underline{-}} \quad 4 \\
 & \underline{\underline{-}} = \underline{\underline{-}} \quad \underline{\underline{-}} = \underline{\underline{-}} \quad - \quad -J \quad \$ \quad \quad \quad - = \underline{\underline{E}} \quad \underline{\underline{-}} \quad -J = \underline{\underline{E}} \quad \underline{\underline{-}} \\
 & \underline{\underline{-}} = \underline{\underline{-}} \quad \underline{\underline{-}} = \underline{\underline{-}} \quad \left(\underline{\underline{E}} \quad \underline{\underline{-}} \right) = \left(\underline{\underline{E}} \quad \underline{\underline{-}} \right) = \underline{\underline{-}} \quad \left[\underline{\underline{E}} \quad \underline{\underline{E}} \right] \quad \underline{\underline{-}}
 \end{aligned}$$

, # \$ " " \$ = " \$ = J O \$ \$ *

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$$\begin{matrix} \underline{\underline{=}} & \underline{\underline{=}} & \underline{\underline{E}} & \underline{\underline{E}} \\ \underline{\underline{=}} & \underline{\underline{=}} & \underline{\underline{=}} & \underline{\underline{=}} \\ \underline{\underline{=}} & \underline{\underline{=}} & \underline{\underline{=}} & \underline{\underline{=}} \end{matrix}$$

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IV.2. Déformation dans une direction , allongement unitaire dans une direction, glissement de deux directions orthogonales

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$$\begin{pmatrix} - & - & - & \underline{\underline{=}} & \underline{\underline{=}} & = & \underline{\underline{=}} & = & \underline{\underline{=}} \end{pmatrix}$$

$$\| \underline{\underline{-}} \| = \| \underline{\underline{-}} \| = \| \underline{\underline{-}} \| = \underline{\underline{=}} = \underline{\underline{=}}$$

$$\| \underline{\underline{-}} \| = \| \underline{\underline{-}} \| = \left(\begin{matrix} 5 & -5 \end{matrix} \right) = \left(\begin{matrix} 5 & -5 \end{matrix} \right) = 5 \left(\begin{matrix} -5 & -5 \end{matrix} \right)$$

$$\frac{\| \underline{\underline{-}} \|}{5} = \left(\begin{matrix} -5 & -5 \end{matrix} \right)$$

O λ^* % 4

$$\lambda = \frac{-}{5} = + \frac{-5}{5} = + \left(\begin{matrix} -5 & -5 \end{matrix} \right)$$

√ =

Définition

\vec{u} \vec{v} \vec{w} \vec{x} \vec{y} \vec{z}
 $\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos(\theta)$ $\vec{u} \cdot \vec{w} = \|\vec{u}\| \|\vec{w}\| \cos(\theta')$ $\vec{u} \cdot \vec{x} = \|\vec{u}\| \|\vec{x}\| \cos(\theta'')$ $\vec{u} \cdot \vec{y} = \|\vec{u}\| \|\vec{y}\| \cos(\theta''')$ $\vec{u} \cdot \vec{z} = \|\vec{u}\| \|\vec{z}\| \cos(\theta''')$

Définition de l'allongement unitaire dans la direction \vec{u}

$$\delta(\vec{u}) = \frac{\|\vec{u}\|}{\|\vec{u}\|} = 1$$

$\lambda(\vec{u}) = \sqrt{1 + \frac{(\vec{u} \cdot \vec{u})^2}{\|\vec{u}\|^4}}$ $\lambda(\vec{u}) = \sqrt{1 + \frac{(\vec{u} \cdot \vec{u})^2}{\|\vec{u}\|^4}}$

$$\delta(\vec{u}) = \frac{\|\vec{u}\|}{\|\vec{u}\|} = 1$$

\$

$$\delta(\vec{u}) = \lambda(\vec{u}) = \sqrt{1 + \frac{(\vec{u} \cdot \vec{u})^2}{\|\vec{u}\|^4}}$$

7 H8

$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos(\theta)$ $\vec{u} \cdot \vec{w} = \|\vec{u}\| \|\vec{w}\| \cos(\theta')$ $\vec{u} \cdot \vec{x} = \|\vec{u}\| \|\vec{x}\| \cos(\theta'')$ $\vec{u} \cdot \vec{y} = \|\vec{u}\| \|\vec{y}\| \cos(\theta''')$ $\vec{u} \cdot \vec{z} = \|\vec{u}\| \|\vec{z}\| \cos(\theta''')$

$$\alpha = \frac{\|\vec{u}\|}{\|\vec{u}\|} = \frac{\|\vec{u}\|}{\|\vec{u}\|}$$

$\lambda(\vec{u}) = \sqrt{1 + \frac{(\vec{u} \cdot \vec{u})^2}{\|\vec{u}\|^4}}$ $\lambda(\vec{u}) = \sqrt{1 + \frac{(\vec{u} \cdot \vec{u})^2}{\|\vec{u}\|^4}}$ $\lambda(\vec{u}) = \sqrt{1 + \frac{(\vec{u} \cdot \vec{u})^2}{\|\vec{u}\|^4}}$

$$\alpha = \frac{-5 \cdot 0^{-1}}{\sqrt{-5 \cdot 0^{-1}} \sqrt{-1 \cdot 0^{-1}}}$$

, # 7 8 ') , = \$
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$$\alpha = \frac{-5 \left(\frac{-1}{5} \right)^{-1}}{\sqrt{-5 \left(\frac{-1}{5} \right)^{-1}} \sqrt{-1 \left(\frac{-1}{5} \right)^{-1}}} = \frac{-5 \frac{-1}{5} + \frac{-1}{5}}{\sqrt{+ \frac{-1}{5} \frac{-1}{5}} \sqrt{+ \frac{-1}{5} \frac{-1}{5}}}; \quad -5 \frac{-1}{5} = 5\% \quad \$ 4$$

$$\alpha = \frac{-5 \frac{-1}{5}}{\sqrt{+ \frac{-1}{5} \frac{-1}{5}} \sqrt{+ \frac{-1}{5} \frac{-1}{5}}}$$

7 58

\$ \$ A " \$ -5 -1/5 \$ * " \$ 3
 \$ \$ % -5 = - -1/5 = -? 7 ≠ 78 4

$$\alpha = \frac{- \frac{-1}{5}}{\sqrt{+ \frac{-1}{5} \frac{-1}{5}} \sqrt{+ \frac{-1}{5} \frac{-1}{5}}} = \frac{?}{\sqrt{+ \frac{-1}{5} \frac{-1}{5}} \sqrt{+ \frac{-1}{5} \frac{-1}{5}}}$$

A # \$

*

V. FORMULATION EN FONCTION DES DEPLACEMENTS

V.1. Tenseur gradient du déplacement

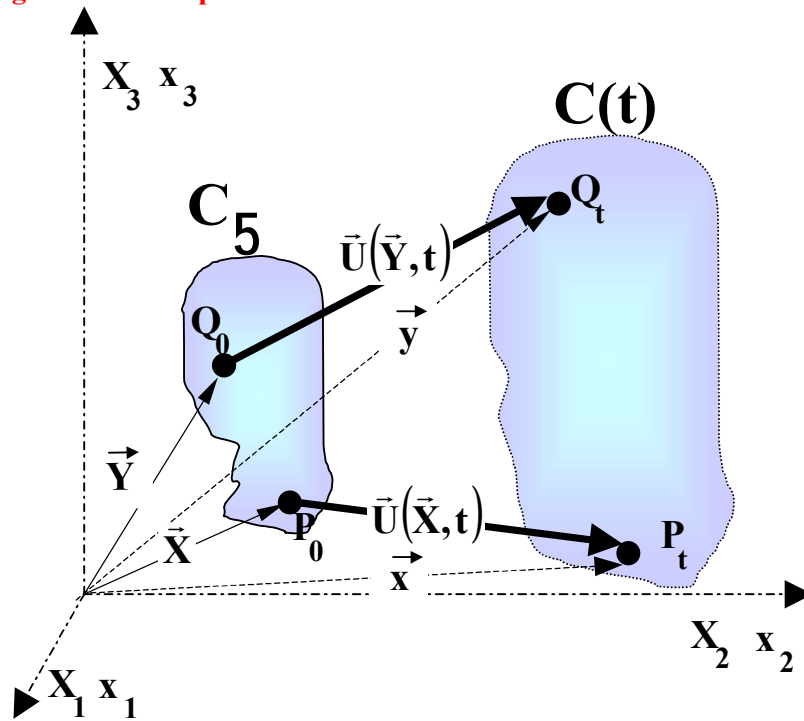


Figure 10: Configuration initiale C_0 et actuelle $C(t)$ d'un solide avec représentation des vecteurs déplacements.

;

$$\bar{\mathbf{1}}(\bar{\mathbf{x}}) = \bar{\mathbf{x}}_5 \bar{\mathbf{x}} = ; \bar{\mathbf{x}} - ; \bar{\mathbf{x}}_5 = \bar{\mathbf{7}} \bar{\mathbf{x}} \bar{\mathbf{8}} - \bar{\mathbf{3}}$$

,

$$\underline{\underline{H}} = \overline{\text{grad}} [\bar{U}(\bar{X}^i)] \text{ soit } H_{ij} = \frac{\partial U_i}{\partial X_j}$$

7 8

,

$$\bar{\mathbf{1}}(\bar{\mathbf{x}}) = \bar{\mathbf{7}} \bar{\mathbf{x}} \bar{\mathbf{8}} - \bar{\mathbf{3}}$$

E 4

$$\underline{\underline{Q}} = \underline{\underline{E}} -$$

7 8

Expression du tenseur des déformations de Green-Lagrange $\underline{\underline{\epsilon}}$ en fonction du tenseur gradient des déplacements $\underline{\underline{u}}$

$$\underline{\underline{\epsilon}} = \frac{1}{2} (\underline{\underline{u}} + \underline{\underline{u}}^T)$$

$$\underline{\underline{\epsilon}} = \frac{1}{2} (\underline{\underline{u}} + \underline{\underline{u}}^T) = \underline{\underline{\epsilon}} + \frac{1}{2} (\underline{\underline{u}} - \underline{\underline{u}}^T)$$

7 8

V.2. Décomposition du déplacement

$$\underline{\underline{u}} = \underline{\underline{\epsilon}} + \underline{\underline{\omega}}$$

$$\underline{\underline{\omega}} = \frac{1}{2} (\underline{\underline{u}} - \underline{\underline{u}}^T)$$

$$\underline{\underline{\omega}} = \mu \underline{\underline{\omega}}$$

$$\underline{\underline{\omega}} = -\frac{1}{2} (\underline{\underline{u}} - \underline{\underline{u}}^T) = \left(\frac{\underline{\underline{R}}}{2} \right) \underline{\underline{\omega}}$$

$$\underline{\underline{\omega}} = \frac{1}{2} (\underline{\underline{u}} - \underline{\underline{u}}^T)$$

$$\underline{\underline{\omega}} = \mu \underline{\underline{\omega}} \quad (\mu = \text{rot})$$

$$\underline{\underline{\omega}} = \underline{\underline{R}} \underline{\underline{\omega}} = \underline{\underline{R}} \underline{\underline{\omega}} \quad (\mu = \text{rot})$$

$$\underline{\underline{\omega}} = \frac{1}{2} (\underline{\underline{u}} - \underline{\underline{u}}^T)$$

$$\lambda(\underline{\underline{\omega}}) = \sqrt{\underline{\underline{\omega}} \cdot \underline{\underline{\omega}}} = \underline{\underline{R}} \underline{\underline{\omega}} = \sqrt{\underline{\underline{R}}} \quad (\mu = \text{rot})$$

$$\underline{\underline{\omega}} = \underline{\underline{E}} \underline{\underline{\omega}} \quad (\mu = \text{rot})$$

$$\underline{\underline{\omega}} \cdot \underline{\underline{\omega}} = \underline{\underline{E}} \cdot \underline{\underline{E}} \underline{\underline{\omega}} \cdot \underline{\underline{\omega}}$$

$$\underline{\underline{\omega}} \cdot \underline{\underline{\omega}} = \underline{\underline{R}} \underline{\underline{\omega}} \cdot \underline{\underline{\omega}}$$

$$\delta = \delta(\vec{\gamma}) = \lambda - = \sqrt{R} - = \sqrt{+} -$$

7 G8

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 # & \$ \$ \$ %\$ \$ \$
 % " \$ # 3 & 3 \$
 " 4

$$\bar{3} = \underline{\underline{E}} \vec{\gamma} \vec{\gamma}$$

O % " \$ * % \$
 I , = \$ \$
 \$ % 3 { \vec{\gamma} \% \vec{\gamma} \% \vec{\gamma} } 3 { \bar{3} \% \bar{3} \% \bar{3} } %
 : O# \$ % #A
 = , 1 3 \$ \$
 " \$ \vec{\gamma} " \$ \$ *) : * \$
 \bar{3} , \$ 1 \$ \$ \$;
1 \$ * , +
 % \$ \$ \$ \$ O
 \$ " \$ \bar{3} = \underline{\underline{E}} \vec{\gamma} / #
 : \$ % 1 + : " \lambda = \| \bar{3} \| = \sqrt{R} O
 \$ \$ \$ * \$
 \$ % \$ 4

$$\underline{\underline{=}} = \underline{\underline{=}} = \underline{\underline{=}}$$

7 H8

O ? \$ " T * U
 T * \$ U 1 + ; T * U

T * \$ U, \$ 1 3 \$ { \bar{1} \bar{2} \bar{3} }
4

$$\underline{\underline{1}} = \begin{bmatrix} \lambda & 5 & 5 \\ 5 & \lambda & 5 \\ 5 & 5 & \lambda \end{bmatrix}$$

O# : \$ \$ + 3 { \bar{3} \bar{3} \bar{3} } 1
" \$ 3 \$ " \$ \$ O %
" \$ \$ \$ " \$ " \$ #
\$ 1 " \$ = \$ \$ 3
*

, O \$ O' # \$ \underline{\underline{O}} = \underline{\underline{E}} \underline{\underline{E}} \$
= \underline{\underline{1}} \% \underline{\underline{O}} # \$ 4

$$\underline{\underline{O}} = (\underline{\underline{.1}}) \underline{\underline{.1}} = \underline{\underline{1}} \underline{\underline{.1}} = \underline{\underline{.1}}$$

; = , # " = \underline{\underline{1}} \underline{\underline{1}} = \underline{\underline{1}} \underline{\underline{1}} , O \$ O' \underline{\underline{1}}
\$ \$ \underline{\underline{1}} 4 \underline{\underline{O}}

$$\underline{\underline{C}} = \underline{\underline{U}} \underline{\underline{U}} = \underline{\underline{U}}$$

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, \$), \$
\$ 4

$$\underline{\underline{E}} = -(\underline{\underline{U}} - \underline{\underline{I}})$$

7 8

$$\begin{matrix}
 3 & \$ & \{ \bar{7} \% \bar{7} \% \bar{7} \% \} \% & \$ & \underline{\underline{0}} \% = & \underline{\underline{1}} \\
 , & \$ & \underline{\underline{+}} & 3 & \{ \bar{3} \% \bar{3} \% \bar{3} \% \}
 \end{matrix}$$

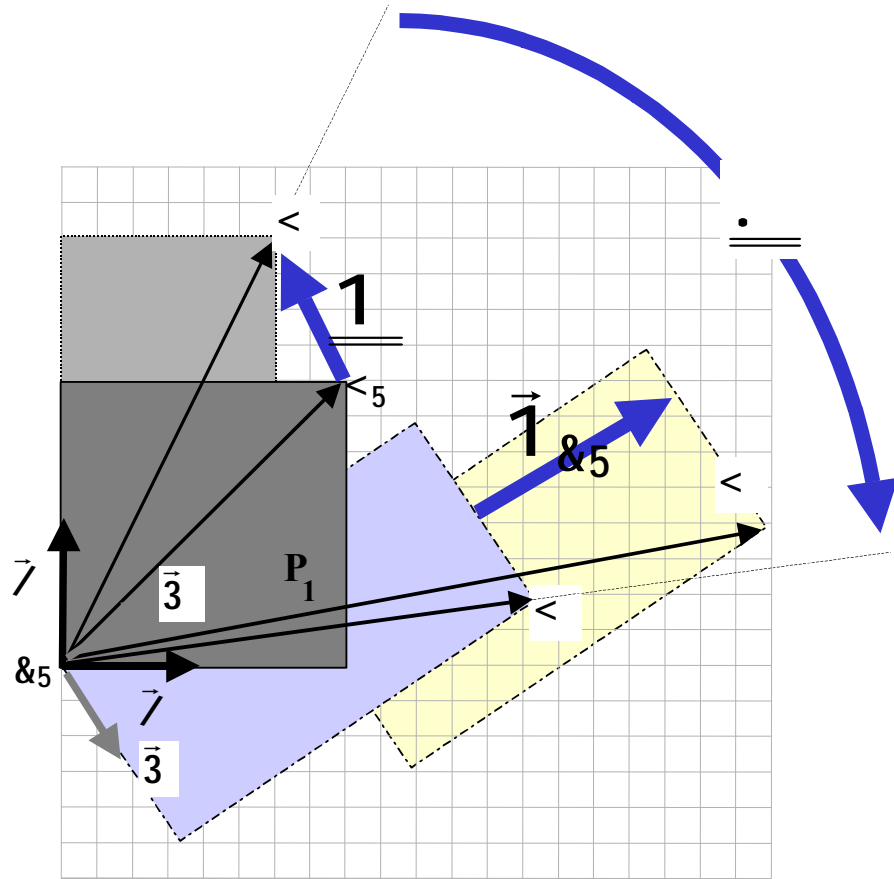


Figure 11: Signification géométrique de la décomposition RU.

$$\begin{matrix}
 0 & & \&5 & <_5 & \$ & & \$ & \# \\
 \$ & & \$ & & \underline{\underline{=}} = \overline{\&5 <_5} , & & \underline{\underline{=}} & \# & \$ & \bar{\Phi} \\
 \$ & \$ & 3 & \$ & \$ & & \underline{\underline{=}} = \overline{\& <_5} = \underline{\underline{E}} \underline{\underline{=}} & \underline{\underline{=}} & , \# & \$ \\
 \underline{\underline{1}} & <_5 & < & \&5 & \$ & , \# & " \$ & \overline{\&5 <_5} & \$ \\
 " \$ & \overline{\&5 <_5} & 2 & \# & " \$ & 3 & \bar{7} & \bar{7} & \# & \$ \\
 \# & \$ & \underline{\underline{1}} & , \# & \$ & \underline{\underline{=}} & < < & \&5 & \$ & , & \underline{\underline{=}} \% \\
 \# & & " \$ & \overline{\&5 <_5} & \$ & " \$ & \overline{\&5 <_5} & \% & & & \\
 \$ & , & & " \$ & 3 & \bar{7} & \bar{7} & " \$ & & &
 \end{matrix}$$

3 \$ 3 3 " \$ 1 & 5 7
8 & 5 & < <

O \$ \$ # " \$ 3
\$ \$ " : \$ " , \$
4

$$\bar{\Phi} = \underline{\underline{.78}} + \bar{L}78$$

' , # *
\$ - # \$ " 4

$$\underline{\underline{E}} = \underline{\underline{.}} \rightarrow \underline{\underline{E}} = \underline{\underline{E}}^-$$

$$\underline{\underline{O}} = \underline{\underline{E}} \underline{\underline{E}} = \underline{\underline{=}}$$

$$\underline{\underline{/}} = \underline{\underline{E}} \underline{\underline{E}} = \underline{\underline{=}}$$

$$\underline{\underline{=}} = \underline{\underline{E}} \underline{\underline{E}} = \underline{\underline{=}}$$

VI DEFORMATION EN PETITE TRANSFORMATION

VI.1. Petites transformations, petites déformations, petits déplacements

$$\underline{\underline{O}} = \underline{\underline{u}} + \underline{\underline{\varepsilon}} + \underline{\underline{Q}} \underline{\underline{Q}}$$

$$\underline{\underline{u}} = \underline{\underline{\varepsilon}} + \underline{\underline{Q}} \underline{\underline{Q}}$$

6 \$ $\underline{\underline{Q}} \underline{\underline{Q}} = \partial_1 \partial_1 \underline{\underline{Q}} \underline{\underline{Q}}$ " # 7\$ 3

8% % # $\underline{\underline{Q}}$ =

% $\|\underline{\underline{Q}}\|$

$$\eta = \|\underline{\underline{Q}}\| = \sqrt{\sum_{\%} \underline{\underline{Q}}}$$

; " # (# \$ \$

\$; " \$

\$ " , \$ \$ (,

(\$ 7 \$ % \$

8 # # ($\underline{\underline{Q}} \underline{\underline{Q}}$ 4 $\underline{\underline{Q}}$

3 \$

VI.2. Expression des tenseurs C et E sous l'hypothèse HPP

$$\underline{\underline{O}} = \underline{\underline{u}} + \underline{\underline{\varepsilon}} + \underline{\underline{Q}} \underline{\underline{Q}} \Rightarrow \underline{\underline{O}} \cong \underline{\underline{u}} + \underline{\underline{\varepsilon}} \quad 7 \ 8$$

$$\underline{\underline{u}} = \underline{\underline{\varepsilon}} + \underline{\underline{Q}} \underline{\underline{Q}} \Rightarrow \underline{\underline{u}} \cong \underline{\underline{\varepsilon}} \quad 7 \ 8$$

$$\lambda = \frac{\delta}{4} = \frac{\sqrt{5}}{4} \Rightarrow \delta = \sqrt{5}$$

$$\lambda = \frac{\delta}{4} = \frac{\sqrt{5}}{4} \Rightarrow \delta = \sqrt{5}$$

$$\lambda = \frac{\delta}{4} = \frac{\sqrt{5}}{4} \Rightarrow \delta = \sqrt{5}$$

$$\lambda = \frac{\delta}{4} = \frac{\sqrt{5}}{4} \Rightarrow \delta = \sqrt{5}$$

7 8

$$\delta = \sqrt{5}$$

7 8

$$\lambda = \frac{\delta}{4} = \frac{\sqrt{5}}{4} \Rightarrow \delta = \sqrt{5}$$

VI.3. Conditions de compatibilité

$$\lambda = \frac{\delta}{4} = \frac{\sqrt{5}}{4} \Rightarrow \delta = \sqrt{5}$$

$$\varepsilon ? = - \left(\frac{\partial \mathbf{1}}{\partial = ?} + \frac{\partial \mathbf{1} ?}{\partial =} \right)$$

, \$ # \$ " \$ " 4 \$ \$

$$Q ? = \frac{\partial \mathbf{1}}{\partial = ?} \quad \omega ? = - \left(\frac{\partial \mathbf{1}}{\partial = ?} - \frac{\partial \mathbf{1} ?}{\partial =} \right)$$

, " \$ % \$ " \$ \$

$$\frac{\partial \omega ?}{\partial = \mathbf{M}} = - \left(\frac{\partial \mathbf{1}}{\partial = \mathcal{P} = \mathbf{M}} - \frac{\partial \mathbf{1} ?}{\partial = \partial = \mathbf{M}} \right)$$

? ($(\partial \mathbf{1}_{\mathbf{M}} @ \partial = \partial = ?) @ \% \#$ \$ # \$

$$\frac{\partial \omega ?}{\partial = \mathbf{M}} = - \left(\frac{\partial \mathbf{1}}{\partial = \mathcal{P} = \mathbf{M}} + \frac{\partial \mathbf{1}_{\mathbf{M}}}{\partial = \partial = ?} \right) - \left(\frac{\partial \mathbf{1}_{\mathbf{M}}}{\partial = \partial = ?} + \frac{\partial \mathbf{1} ?}{\partial = \partial = \mathbf{M}} \right)$$

\$ \$ \$ W " 3 -
" " # " K "
\$ 4

$$\frac{\partial \omega ?}{\partial = \mathbf{M}} = \frac{\partial}{\partial = ?} \left\{ - \left(\frac{\partial \mathbf{1}}{\partial = \mathbf{M}} + \frac{\partial \mathbf{1}_{\mathbf{M}}}{\partial =} \right) \right\} - \frac{\partial}{\partial =} \left\{ - \left(\frac{\partial \mathbf{1}_{\mathbf{M}}}{\partial = ?} + \frac{\partial \mathbf{1} ?}{\partial = \mathbf{M}} \right) \right\} \quad \frac{\partial \omega ?}{\partial = \mathbf{M}} = \frac{\partial \varepsilon_{\mathbf{M}}}{\partial = ?} - \frac{\partial \varepsilon_{? \mathbf{M}}}{\partial =}$$

6 $(\underline{\underline{\varepsilon}} = 5)$ % $\underline{\underline{\omega}}$ \$
 \$ $\underline{\underline{Q}}$ $\underline{\underline{\omega}}$ 4

$$\underline{\underline{\varepsilon}} = 5 \Rightarrow \underline{\underline{\omega}} = \$ \underline{\hspace{2cm}} \% \underline{\underline{Q}} = \underline{\underline{\omega}}$$

, \$ \$ \$ * $\bar{1}_5$ \$

$$Q_? = \frac{\partial 1}{\partial = ?} = \omega_?() \Rightarrow 1 = \omega_?() = ? + 1_5$$

- % \$ \$ \$ \$ \$ *

- " # \$ \$ 3 % # \$ \$

$$\frac{\partial \omega_?}{\partial = M} = \frac{\partial \varepsilon_M}{\partial = ?} - \frac{\partial \varepsilon_{?M}}{\partial =}$$

7 ! 8

1 \$ \$ (3

$$\frac{\partial \omega_?}{\partial = M \partial =} = \frac{\partial \omega_?}{\partial = \partial = M} \neq M8$$

, # \$ \$

$$\frac{\partial \varepsilon_M}{\partial = \partial =} - \frac{\partial \varepsilon_{?M}}{\partial = \partial = ?} = \frac{\partial \varepsilon}{\partial = \partial = M} - \frac{\partial \varepsilon_?}{\partial = \partial = M}$$

7 F 8

O (% ! * ,
 \$ \$ (# 7 ! 8 3 ;
 \$ $\underline{\underline{\omega}}$ # \$ K $\underline{\underline{\varepsilon}}$ - %
 \$ $\underline{\underline{Q}}$, #
 \$ * \$ \$ - (

$$\frac{\partial \mathbf{1}}{\partial = ?} = \varepsilon ? + \omega ?$$

$$3 \quad X, \quad " \quad \frac{\partial \mathbf{1}}{\partial = \varrho =} = \frac{\partial \mathbf{1}}{\partial = \partial = ?} \neq \mathfrak{B}$$

$$\frac{\partial}{\partial =} (\varepsilon ? + \omega ?) = \frac{\partial}{\partial = ?} (\varepsilon + \omega) \neq \mathfrak{B}$$

; \$ 3 # 7 ! 8 , Q * Q #
 \$ \$ \$ 3 , !
 7 F 8 " # \$ 4

$$\frac{\partial \varepsilon}{\partial = \partial =} = \frac{\partial}{\partial =} \left\{ \frac{\partial \varepsilon}{\partial =} + \frac{\partial \varepsilon}{\partial =} - \frac{\partial \varepsilon}{\partial =} \right\}$$

$$\frac{\partial \varepsilon}{\partial = \partial =} = \frac{\partial}{\partial =} \left\{ \frac{\partial \varepsilon}{\partial =} + \frac{\partial \varepsilon}{\partial =} - \frac{\partial \varepsilon}{\partial =} \right\}$$

$$\frac{\partial \varepsilon}{\partial = \partial =} = \frac{\partial}{\partial =} \left\{ \frac{\partial \varepsilon}{\partial =} + \frac{\partial \varepsilon}{\partial =} - \frac{\partial \varepsilon}{\partial =} \right\}$$

$$\frac{\partial \varepsilon}{\partial = \partial =} = \frac{\partial \varepsilon}{\partial =} + \frac{\partial \varepsilon}{\partial =}$$

$$\frac{\partial \varepsilon}{\partial = \partial =} = \frac{\partial \varepsilon}{\partial =} + \frac{\partial \varepsilon}{\partial =}$$

$$\frac{\partial \varepsilon}{\partial = \partial =} = \frac{\partial \varepsilon}{\partial =} + \frac{\partial \varepsilon}{\partial =}$$

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 # (3 ' 33 \$ # %
 \$ 3 # \$ " 4

$$\underbrace{\frac{\partial}{\partial x_k \partial x_k} \varepsilon_{ij}}_{\Delta \varepsilon_{ij}} + \underbrace{\frac{\partial}{\partial x_i \partial x_j} \overbrace{\varepsilon_{kk}}^{\text{trace}(\underline{\varepsilon})}}_{\text{gr} \ddot{a}d\{\text{gr} \ddot{a}d[\text{trace}(\underline{\varepsilon})]\}} - \underbrace{\frac{\partial}{\partial x_j} \left(\frac{\partial}{\partial x_k} \varepsilon_{ik} \right)}_{\text{gr} \ddot{a}d[\text{div}(\underline{\varepsilon})]} - \frac{\partial}{\partial x_i \partial x_k} \varepsilon_{jk} = 5$$

O \$ * # " ' 33 4

$$\Delta(\underline{\varepsilon}) + \text{gr} \ddot{a}d \{ \text{gr} \ddot{a}d [\text{trace}(\underline{\varepsilon})] \} = \text{gr} \ddot{a}d [\text{div}(\underline{\varepsilon})] + \{ \text{gr} \ddot{a}d [\text{div}(\underline{\varepsilon})] \}^T$$

7 H 8

VII VITESSES DE DEFORMATION

VII.1. Tenseur gradient des vitesses $\underline{\underline{L}}$

$$\underline{\underline{L}} = \frac{\partial \underline{\underline{v}}}{\partial \underline{\underline{x}}}$$

$$\underline{\underline{L}} = \frac{\partial \underline{\underline{v}}}{\partial \underline{\underline{x}}}$$

7 58

$$\underline{\underline{L}} = \frac{\partial \underline{\underline{v}}}{\partial \underline{\underline{x}}}$$

$$\underline{\underline{L}} = \frac{\partial \underline{\underline{v}}}{\partial \underline{\underline{x}}}$$

7 8

$$\underline{\underline{L}} = \frac{\partial \underline{\underline{v}}}{\partial \underline{\underline{x}}}$$

VII.2. Variation temporelle du tenseur gradient de la transformation $\underline{\underline{\dot{E}}} = \frac{\partial \underline{\underline{E}}}{\partial t}$

$$\underline{\underline{\dot{E}}} = \frac{\partial \underline{\underline{E}}}{\partial t}$$

$$\underline{\underline{\dot{E}}} = \frac{\partial \underline{\underline{E}}}{\partial t}$$

; # \$ " " : 3 \$ * " # \$
 " \$ -4

$$\dot{\underline{\underline{\epsilon}}} = \frac{\partial(\underline{\underline{\epsilon}})}{\partial t} = \frac{\partial(\underline{\underline{\epsilon}})}{\partial t} = \frac{\partial(\underline{\underline{\epsilon}})}{\partial t} = \frac{\partial(\underline{\underline{\epsilon}})}{\partial t} = \frac{\partial(\underline{\underline{\epsilon}})}{\partial t} = \dot{\underline{\underline{\epsilon}}}$$

$$\dot{\underline{\underline{\epsilon}}} = \underline{\underline{\epsilon}}$$

7 8

VII.3. Tenseur vitesse de déformation et vitesse de rotation

$$\dot{\underline{\underline{\epsilon}}} = -(\underline{\underline{\epsilon}} + \underline{\underline{\epsilon}}) = -\left(\left(\frac{\partial''}{\partial t} \right) + \left(\frac{\partial''}{\partial t} \right) \right)$$

7 8

$$\dot{\epsilon}_{?} = -(\frac{?}{\partial} + \frac{?}{\partial}) = -\left(\frac{\partial''}{\partial} + \frac{\partial''}{\partial} \right)$$

7 38

$$\dot{\underline{\underline{\omega}}} = -(\underline{\underline{\omega}} - \underline{\underline{\omega}}) = -\left(\left(\frac{\partial''}{\partial t} \right) - \left(\frac{\partial''}{\partial t} \right) \right)$$

7 8

$$\dot{\omega}_{?} = -(\frac{?}{\partial} - \frac{?}{\partial}) = -\left(\frac{\partial''}{\partial} - \frac{\partial''}{\partial} \right)$$

7 38

