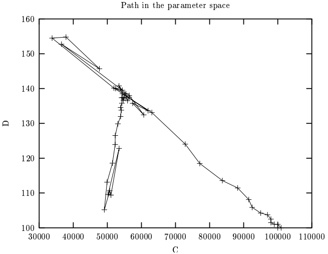
Leçon 15 : Stratégies d’identification



- Mise en place du problème
- Différentes classes d’algorithmes
- Exemple
- Recommandations pratiques

Remarks on the model implementation

- Validity domain
- Which model is to be used ?
  - Compromise between effort to identify and precision
  - Compromise between numerical efficiency and price of the development
  - Size of the domain *versus* complexity
- Adapt the model to the experimental data base
- Adapt the model to the application

Methodology of the numerical identification of the material parameters

- Type of variables
  - Observable variables, internal variables, material parameters
- Some observable variables are prescribed ; identification is performed by trying to fit the simulation and the experimental data on *the other* observable variables
- E.g., for a test under ε control, the comparison is made on stresses ; several responses can be used (multiaxial loadings)
- ALL the tests in the data base must be used (the discrepancy can be eventually computed with a weighting factor)

Classical formulation of the identification problem

- Definition of a *cost function*
- One has to *minimise* this function versus its arguments
- E.g., curve fitting,
  - Experimental response, with a sampling of the response in points at t_i, $z(t_i)$
  - Approximation by means of a simulation $y(\underline{x}, t)$, depending on the parameters $\underline{x}$
  - One has to minimize the following :

$$f(\underline{x}) = \sum_i (y(\underline{x}, t_i) - z(t_i))^2$$

- Often, *constraints* have to be considered on the arguments, then

Find $\underline{x}^*$ **which minimize** $f(\underline{x})$ **with** $g_i(\underline{x}) \leqslant 0$**,** $i = 1...N_c$

Definitions

- *Linear* programming if f and g_i are linear
- *Convexe* programming if f and g_i are convex
- *Non linear* programming if f and g_i have nothing special !
- *Admissible domain* = set of the admissible solutions
- A *global* minimum (optimal solution) minimises the cost function On the whole admissible domain ; this is nothing but $\underline{x}^*$ which realises the minimum of $f(\underline{x})$, respecting the constraints
- A *local* optimum in $\underline{x}^*$ minimises the function in the vicinity of $\underline{x}^*$

Examples

- Engineering of electrical circuits
  - Variables : thickness and length of the circuits
  - Constraints : to know how to make the circuit, duration of the fabrication process
  - Objective : minimum cost and electric power
- Curve fitting for material parameters in constitutive equations
  - Variables : material parameters
  - Constraints : known limitations of the parameters
  - Objective : minimal difference between exp and sim curves

Differentiable functions

- Gradient, formed by means of the first derivatives :

$$\nabla f = \left\{ \frac{\partial f}{\partial x_1} \cdots \frac{\partial f}{\partial x_n} \right\}^T = \left\{ f_{,1} \ldots f_{,n} \right\}^T$$

- Hessian, formed by means of the second derivatives (symmétrical matrix) :

$$\begin{pmatrix} f_{,11} & f_{,12} & \cdots & f_{,1n} \\ f_{,21} & f_{,22} & \cdots & f_{,2n} \\ \cdots & \cdots & \cdots & \cdots \\ f_{,n1} & f_{,n2} & \cdots & f_{,nn} \end{pmatrix}$$

Stationnary point

- If f is differentiable and has an extremal value at point $\underline{x}^*$, then the gradient is null in $\underline{x}^*$ (necessary condition)
- The function is minimum/maximum on the points of the boundary of the admissible domain or at the stationnary points
- To know if a given point is a min or a max, one has to evaluate the matrix H.
  - If the eigenvalues of H are all positive in $\underline{x}^*$, $\underline{x}^*$ realises a *min*
  - If the eigenvalues of H are all negative in $\underline{x}^*$, $\underline{x}^*$ realises a *max*
  - If the eigenvalues of H are negative and positive in $\underline{x}^*$, the case is *critical*
  - If the eigenvalues of H are all equal to zero in $\underline{x}^*$, the case is *undetermined*

If the function f is convex, OK

Minimum and maximum

- To know if a given point is a min or a max, one can also compute the determinant of H, Δ.
  - Si $\Delta(\underline{x}^*) > 0$, $f_{,11}(\underline{x}^*) < 0$, $\underline{x}^*$ realises a *max*
  - Si $\Delta(\underline{x}^*) > 0$, $f_{,11}(\underline{x}^*) > 0$, $\underline{x}^*$ realises a *min*
  - Si $\Delta(\underline{x}^*) < 0$, the case is *critical*
  - Si $\Delta(\underline{x}^*) = 0$, the case is *undetermined*

Example : the function $f(x_1, x_2) = x_1 + \dfrac{1}{x_1} + x_2 + \dfrac{4}{x_2}$ has 4 stationnary points ($x_1 = \pm 1$, $x_2 = \pm 2$). It is minimum at point (1,2), maximum at point (-1, -2), and critical for the two other points.

Various optimisation methods

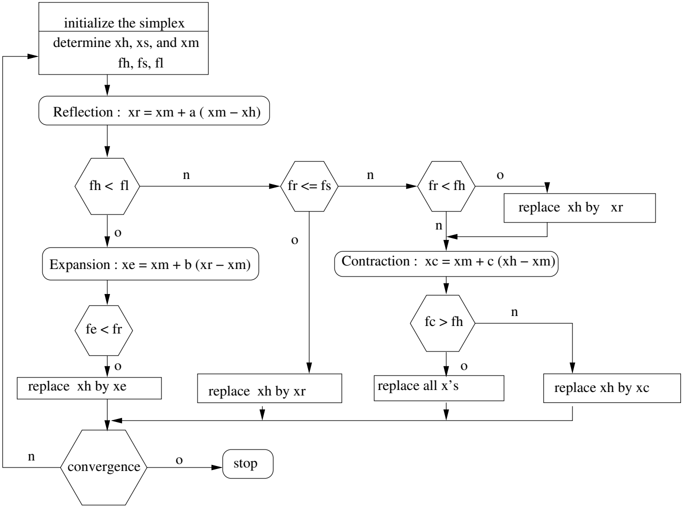
- Order of the method
  - order 0, uses only evaluations of the function, like evolutionnary algorithms, simplex
  - order 1, uses the gradient, descent methods
  - order 2, uses the Hessien, SQP methods

Simplex method (1)

- Order 0, robust, slow with many parameters
- Motion of the points of the "simplex" ($n+1$ if n parameters) :
  - reflexion (from the worst to the best point)
  - expansion (toward the best, in case of success)
  - contraction (toward the barycenter, if the reflection has failed)

J.A. Nelder, R. Mead, Computer Journal*,* **7***, 308–313, 1965*

Simplex method (2)



- xh, xl, xs and xm are respectively the highest, lowest, second lowest points les plus hauts, plus bas, second plus bas point, and the barycenter

Evolutionnary methods

- "Evolutionnary" or "genetic" algorithms to manage a polulation of solutions
- Constraints taken into account by penalization
- Random generation of an initial population
- Progression by creating new indivuduals, and suppressing the worst
- "Normal" generation by crossing two exisiting individual
- Possibility of a "mutation"
- A larger population and a bigger mutation probability increase the random character

Descent methods

- Initial point $\underline{x}^0$
- A sequence is generated, following $\underline{x}^{k+1} = \underline{x}^k + \alpha_k \underline{D}^k$ (with α_k scalar, and $\underline{D}^k$ direction of descent
- *Gradient* method
  - One chooses $\underline{D}^k = -\nabla f(\underline{x}^k)$ (highest slope linie)
  - One optimises α_k, such that $h(\alpha) = f(\underline{x}^k + \alpha \underline{D}^k)$ is minimum
  - STOP iff the difference between two iterations is low (for instance)
  - Good convergence far from the solution, bad convergence in the vicinity of the optimum (zig-zag)

See foe instance this course of Stanford in page 213 and following

Newton method

- Developement at second order in the vicinity of $\underline{x}^k$

$$f(\underline{x}^{k+1}) = f(\underline{x}^k) + {}^T\underline{x}^k \nabla f(\underline{x}^k) + \frac{1}{2}{}^T\underline{x}^k H(\underline{x}^k)\underline{x}^k$$

- One *minimises* the new value of f by choosing :

$$\underline{x}^{k+1} = \underline{x}^k - H(\underline{x}^k)^{-1}\nabla f(\underline{x}^k)$$

- Very good convergence near the optimum (quadratic)

SQP method, sequential quadratic programming (1)

(The vector is no longer underlined $\underline{x}$ to have a clearer notation)

- Order 2 method, which takes into account explicitely the constraints
- x^* realises the minimum of the Lagrange function :

$$L(x) = f(x) + \lambda^T g(x)$$

  in the subspace of the constraints orthogonal to the active constraints
- One minimises a quadratic approximation of the Lagrangian, by neglecting $s^T \nabla g_i$

$$\begin{aligned} \text{minimise } f(x_i) \; + \; s^T \nabla f(x_i) \; + \; \frac{1}{2} s^T H(x_i, \lambda_i) s \qquad & (1) \\ \text{by changing } s \; = \; x_{i+1} - x_i \qquad & (2) \\ \text{so that} \quad g_j(x_i) \; + \; s \nabla g_j(x_i) \; \leq \; 0 \quad , \quad j = 1, N_c \qquad & (3) \end{aligned}$$

SQP method, sequential quadratic programming (2)

One has :

$$H(x_i, \lambda_i) \rightarrow \nabla^2 f(x_i) + \lambda_i^T \nabla^2 g(x_i).$$

One uses a method *QLD* which produces a direction of descent s and a vector of Lagrange multiplyiers.
One finds x_{i+1} by minimizing L along s :

$$x_{i+1} = x_i + \alpha s$$

where α minimises

$$f(x_{i+1}) + \lambda_i^T g(x_{i+1})$$

K. Schittkowski, QLD users's guide, Univ. Bayreuth, Allemagne, 1986

Application to cyclic viscoplasticity

Chosen model

$$\underset{\sim}{\varepsilon} = \underset{\sim}{\varepsilon}^e + \underset{\sim}{\varepsilon}^p$$

$$f(\underset{\sim}{\sigma}, \underset{\sim}{X}, R) = \sqrt{3/2\,(\underset{\sim}{\sigma} - \underset{\sim}{X})^d : (\underset{\sim}{\sigma} - \underset{\sim}{X})^d} - R - \sigma_y$$

$$\dot{\underset{\sim}{\varepsilon}}^p = \dot{p}\frac{\partial f}{\partial \underset{\sim}{\sigma}} = \dot{p}\underset{\sim}{n} \qquad \dot{\underset{\sim}{p}} = \left\langle \frac{f}{K} \right\rangle^n$$

$$\dot{\underset{\sim}{\alpha}} = \dot{\underset{\sim}{\varepsilon}}^p - \frac{3}{2}\frac{C}{D}\underset{\sim}{X}\dot{p}$$

$$R = Q(1 - \exp(-bp)) \qquad \underset{\sim}{X} = \frac{2}{3}C\underset{\sim}{\alpha}$$

The material parameters to identify are the parameters which characterize viscosity, K, n, the parameters of kinematic hardening, C et D, those for isotropic hardening, Q et b, and the initial yield σ_y. The Young's modulus is deduced from the beginning of the tension curve

Expression under onedimensional loading

$$\sigma = X + \eta(R + \sigma_v)$$

with $\eta = 1$ in tension, and $\eta = -1$ in compression.

For isothermal conditions, one has the following expressions :

$$\dot{p} = |\dot{\varepsilon}^p|$$
$$\dot{\alpha} = \dot{\varepsilon}^p - D\alpha\dot{p}$$

and :

$$R = Q(1 - \exp(-bp)) \qquad X = C\alpha$$
$$\sigma_v = K\dot{p}^{1/n}$$

Identification methodology (1)

- **La variable** R sert à rendre compte de l'adoucissement ou du durcissement cyclique, elle évolue peu d'un cycle à l'autre. Par voie de conséquence, le coefficient matériau b est généralement de l'ordre de 10, la déformation plastique cumulée p étant exprimée en mm/mm. La valeur de Q correspond à la différence entre la contrainte maximale au premier cycle et au cycle stabilisé. Elle est positive en cas de durcissement cyclique, et négative en cas d'adoucissement cyclique.
- **La variable** X évolue au contraire rapidement au cours des cycles, pour représenter l'effet Bauschinger. Le coefficient matériau D règle la vitesse de saturation de l'écrouissage, une valeur de 100 correspond à une saturation lente, tandis que 1000 correspond à une saturation plutôt rapide. La valeur asymptotique obtenue en traction simple est (C/D) :

$$X = \frac{C}{D}(1 - \exp(-D\varepsilon^p))$$

Consignes d'identification (2)

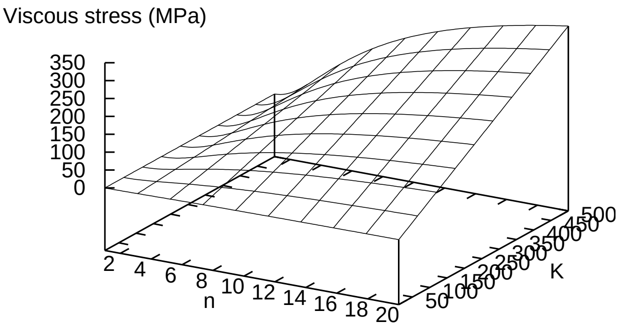
- **La contrainte visqueuse** est d'autant plus grande que l'effet de la vitesse est sensible. Une manière de rendre le comportement insensible à la vitesse de déformation est de prendre une petite valeur de K (avec zéro, on a un modèle de plasticité instantanée). L'exposant n diminue avec la température. On prendrait simplement $n = 1$ pour un matériau purement visqueux à haute température, en annulant d'ailleurs les contraintes internes, les valeurs aux températures intermédiaires peuvent atteindre 20 ou 30 au maximum, avec une variation monotone. Le coefficient K peut varier au contraire de façon non monotone, dans la mesure où il doit être petit à "basse" température, lorsque l'effet de la vitesse est faible, mais également petit à "haute" température, lorsque la contrainte ultime devient faible, la contrainte visqueuse étant bien sûr limitée par celle-ci. On obtient donc souvent des variations "en cloche", les valeurs maximales de K étant obtenues aux températures intermédiaires.

Rôle of each coefficient

R0	σ_y, initial yield stress
Q	cyclic hardening or softening
b	convergence rate to Q
C/ D	asymptotic value of X
D	convergence rate to C/D
K	viscous stress for $\dot{\varepsilon}^p = 1s^{-1}$
n	$\rightarrow$ 1 for high temperature

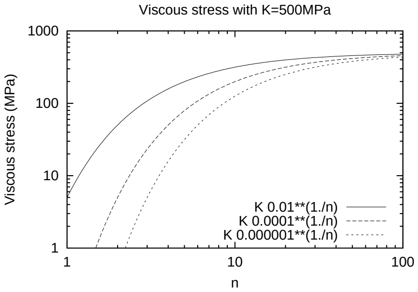
- for $\sigma_y = R = X = 0$, Norton model
- for $\sigma_y = R = 0$, no threshold (non linear viscoelasticity)
- for small K, no more viscous effect ($\rightarrow$ time independent plasticity)

Viscous stress



No strain rate sensitivity when K → 0

Viscous stress



No strain rate sensitivity for high values of n

Phenomenological aspects

- Prediction of R_m (assuming $\dot{\varepsilon}^p \approx \dot{\varepsilon} = 0.001s^{-1}$)
- Prediction of $R_{0.2}$ (assuming $\dot{\varepsilon}^p \approx \dot{\varepsilon} = 0.001s^{-1}$)

$$R_{0.2} = R0 + Q(1 - \exp(-0.002 \times b)) + (C/D)(1 - \exp(-0.002 \times D)) + K \times 0.0$$

- Prediction of the cyclic hardening curve (assuming $\dot{\varepsilon}^p \approx \dot{\varepsilon} = 0.001s^{-1}$)

$$\Delta\sigma/2 = R0 + Q + (C/D)\tanh(D\Delta\varepsilon^p/2) + K \times 0.001^{1/n}$$

- Secondary creep rate

$$\dot{\varepsilon}^p = \left\langle \frac{\sigma - (C/D) - R - R0}{K} \right\rangle^n$$

- Asymptotic stress in relaxation $\sigma_\infty = R0 + Q + (C/D)$

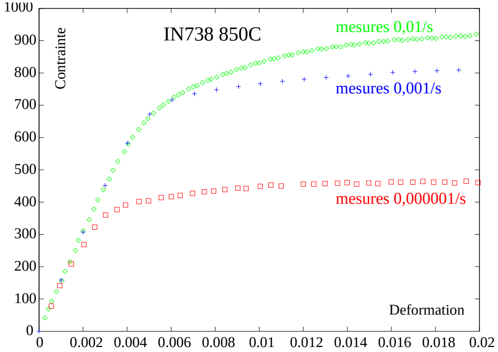
Isotropic and kinematic hardening

```
***behavior gen_evp
**elasticity isotropic young 160000.  poisson 0.3
**potential gen_evp ep
*criterion mises
*flow norton                 K    300.     n    7.
*kinematic linear            C  10000.
*kinematic nonlinear         C 180000.     D  600.
*isotropic nonlinear         R0   300.     Q  100.  b   10
***return
```

$$
\begin{aligned}
\sigma &= R0 + Q(1 - e^{-b\varepsilon^p}) && \textit{isotropic} \\
&+ H\varepsilon^p && \textit{kinematic} \\
&+ C/D\,(1 - e^{-D\varepsilon^p}) && \textit{kinematic} \\
&+ K\,(\dot{\varepsilon}^p)^{1/n} && \textit{viscous}
\end{aligned}
$$

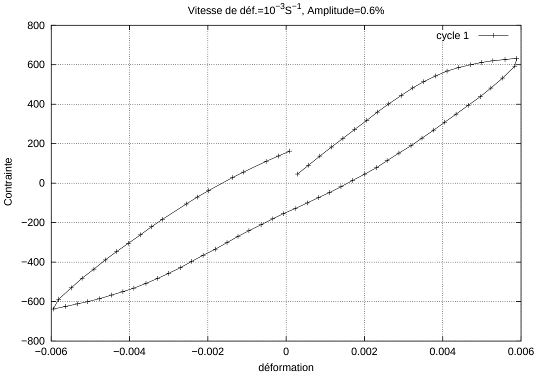
$\rightarrow$ 8 material parameters

Example 1 : BAM data base



- BAM tests, on the Ni-base alloy IN738 at 850°C.
- No noticeable cyclic hardening, then isotropic hardening is taken equal to zero
- Monotonic tests at $\dot{\varepsilon} = 10^{-2}s^{-1}$, $10^{-3}s^{-1}$ $10^{-6}s^{-1}$

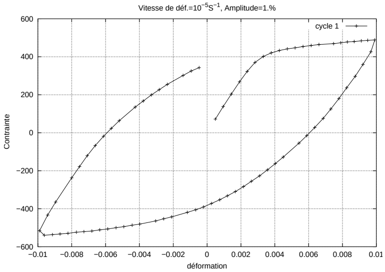
Experimental data base (2)

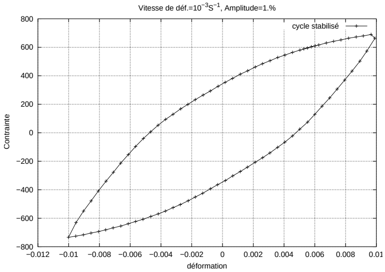


$$\Delta\varepsilon/2 = 0{,}6\ \% \quad \dot{\varepsilon} = 10^{-3}\mathrm{s}^{-1}$$

Cyclic tests, first cycle

Experimental data base (3)





$\Delta\varepsilon/2 = 1\% \quad \dot{\varepsilon} = 10^{-5}s^{-1}$

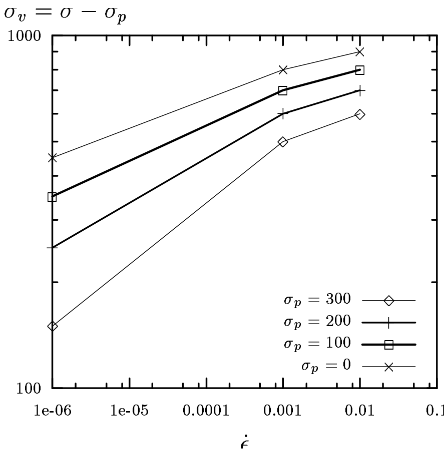
$\Delta\varepsilon/2 = 1\% \quad \dot{\varepsilon} = 10^{-3}s^{-1}$

Cyclic tests, mechanical steady state

Optimization strategy

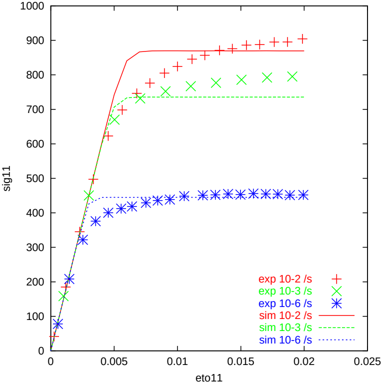
- Estimation des paramètres initiaux, la meilleure possible
- Caractérisation de la contrainte visqueuse sous chargement monotone, avec une partie "plastique" où X et R ne sont pas distingués
- Identification en monotone
- Report de l'indentification en cyclique
- Ajout d'autres essais (fluage, relaxation) pour caractériser la restauration des contraintes internes

Identification without hardening



- "Plastic" and viscous part of the stress $\sigma = \sigma_p + K\dot{\varepsilon}^{1/n}$
- Best n for several choices of σ_p ?
- One keeps σ_p =0, then K is computed

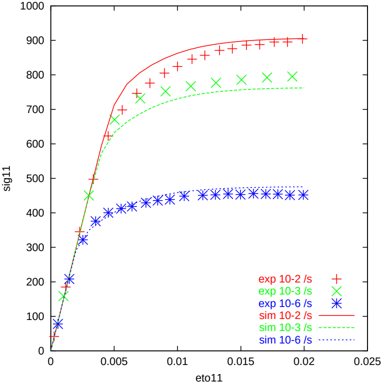
Hardening is now needed...



Simulation with K=1216, n=13.7

- No isotropic hardening
- Cost function = 2323
- The difference exp–sim gives an estimation of kinematic hardening

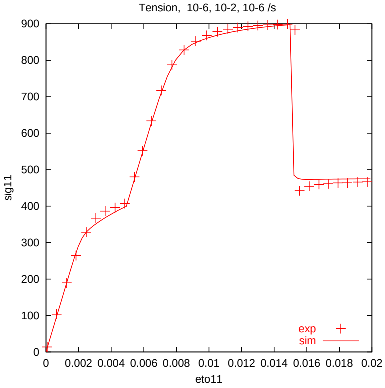
New curve, taking hardening into account

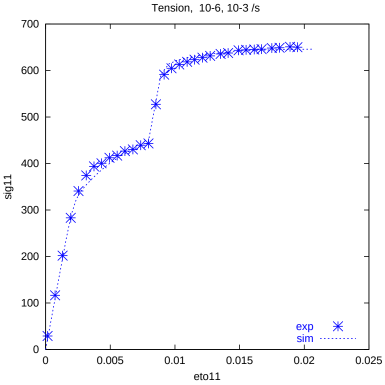


Simulation with K=1150, n=11.0,
C=46500, D=316

- Identification process should be rerun, with free values for K et n
- The new starting point is not as good as the preceding one
- Decreases of the viscous part of the stress, depending on

Other monotonic tests, various strain rates



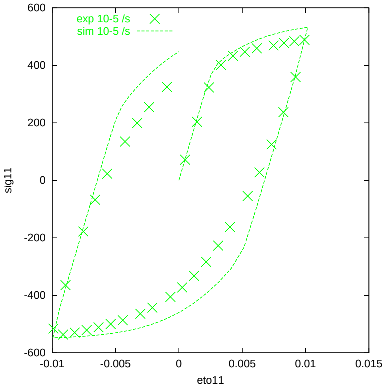


$\dot{\varepsilon} = 10^{-6}s^{-1} - 10^{-2}s^{-1} - 10^{-6}s^{-1}$

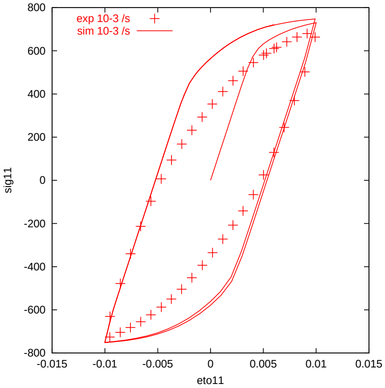
$\dot{\varepsilon} = 10^{-6}s^{-1} - 10^{-3}s^{-1}$

Very good prediction. Normal, since no new information

Coefficients identified with monotonic tests applied to cyclic loadings (1)



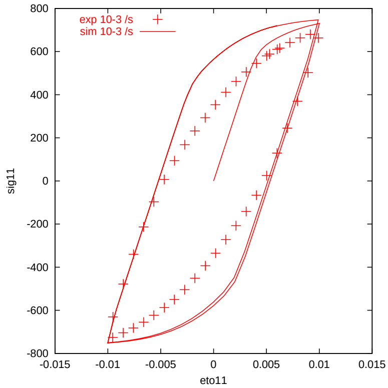
$\Delta\varepsilon/2 = 1\% \quad \dot{\varepsilon} = 10^{-5}s^{-1}$



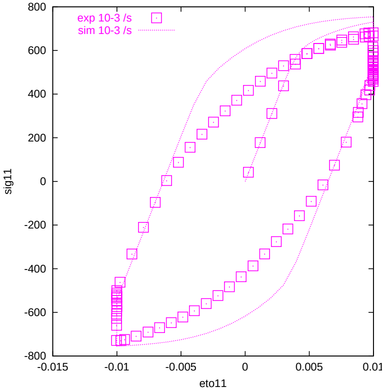
$\Delta\varepsilon/2 = 1\% \quad \dot{\varepsilon} = 10^{-3}s^{-1}$

K=1150, n=11.0, C=46500, D=316

Coefficients identified with monotonic tests applied to cyclic loadings (2)



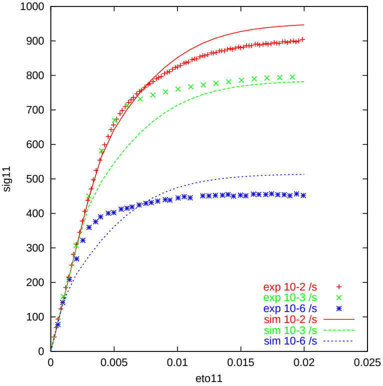
$\Delta\varepsilon/2 = 1\%$ $\dot{\varepsilon} = 10^{-5}s^{-1}$



Cyclic relaxation, 30s T et C

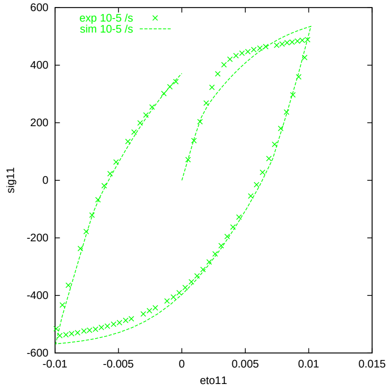
K=1150, n=11.0, C=46500, D=316

Identification using the whole data base (1)

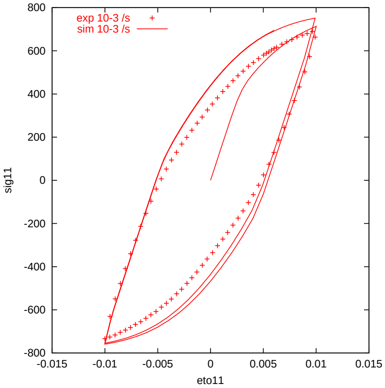


- Compromise between monotonic and cyclic tests
- The model is not perfect, so each series of tests may have a less good fit
- The optimizer finds good results with several values of n

Identification using the whole data base (2)



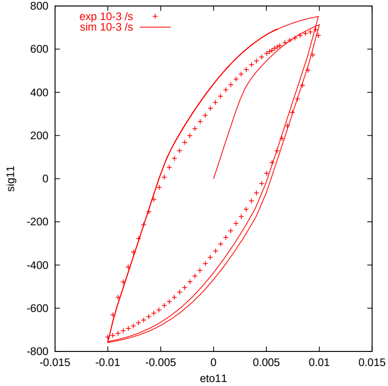
$\Delta\varepsilon/2 = 1\% \quad \dot{\varepsilon} = 10^{-5}s^{-1}$



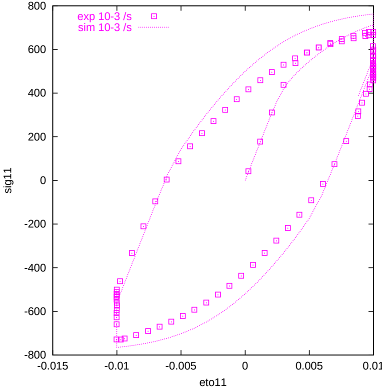
$\Delta\varepsilon/2 = 1\% \quad \dot{\varepsilon} = 10^{-3}s^{-1}$

K=1120, n=7.7, C=104100, D=315

Identification using the whole data base (3)



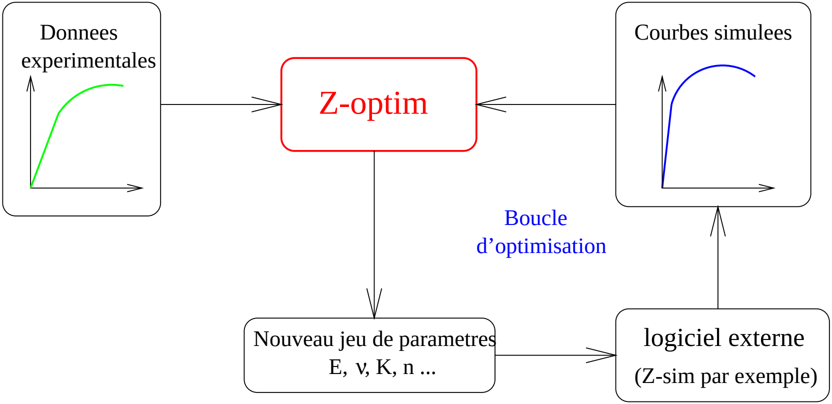
$\Delta\varepsilon/2 = 1\%$ $\dot{\varepsilon} = 10^{-5}s^{-1}$



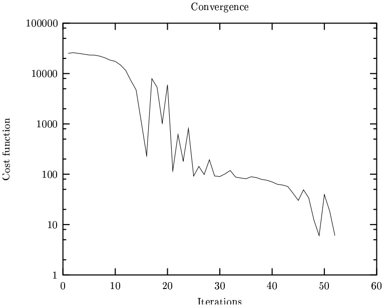
Cyclic relaxation, 30s T et C

K=1120, n=7.7, C=104100, D=315

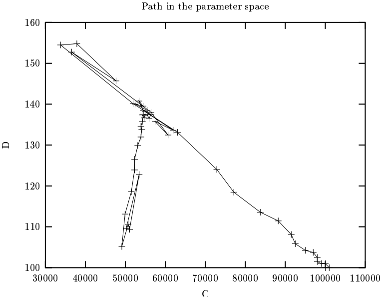
Construction of the optimisation loop



Convergence of the simplex method

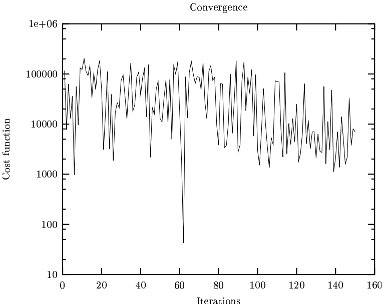


Evolution of the cost function

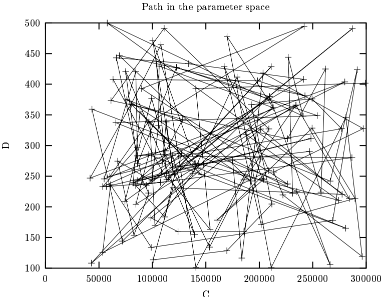


Convergence path

Convergence of a genetic method

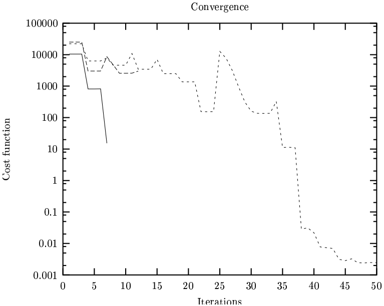


Evolution of the cost function

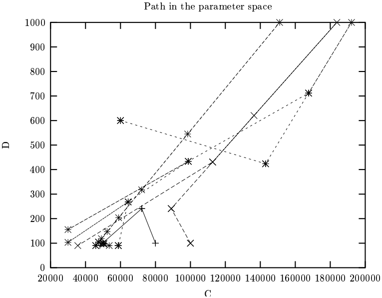


Convergence path

Convergence of the SQP methode



Evolution of the cost function



Convergence path

Exemple 2 : study of the shape of the cost function

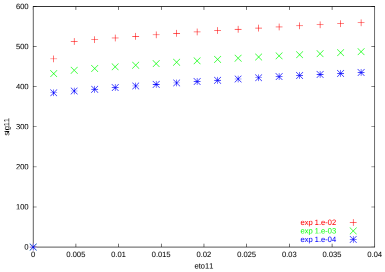
- Monotonic viscoplasticity model :

$$\sigma = R_0 + Q(1 - e^{-bp}) + K\dot{p}^{1/n}$$

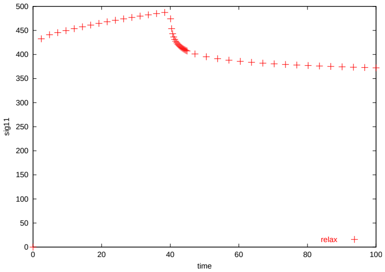
avec $R_0 = 250$, $Q = 100$, $b = 20$, $K = 500$, $n = 7$

- Numerical construction of an "experimental data base" : three hardening tests, under strain control, at $10^{-2}s^{-1}$, $10^{-3}s^{-1}$, $10^{-4}s^{-1}$, and a relaxation test (loading at $\dot{\varepsilon} = 10^{-3}s{-}1$, hold time 60$s$.
- Identification on hardening test only, e.g. at $\dot{\varepsilon} = 10^{-2}s^{-1}$, an other (very good) solution is found, BUT the other tests are not correctly reoresented.
- Identification on all the tests, the good solution is found

Experimental data base under monotonic loading



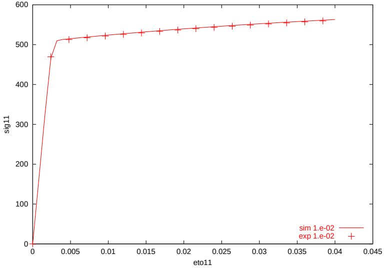
Hardening



Relaxation

$$R_0 = 250, Q = 100, b = 20, K = 500, n = 7$$

Identification on the test at $\dot{\varepsilon} = 10^{-2}s^{-1}$

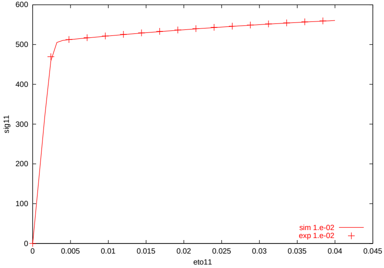


The following points are equivalent :

K	$R0$	Error
250	380	3.98e-06
325	340	2.06e-06
400	300	4.80e-06
425	290	2.97e-06
575	210	2.58e-06
600	200	5.55e-06
675	160	2.84e-06
750	120	6.28e-06

Reference	$R_0 = 250, Q = 100, b = 20, K = 500, n = 7$
Present calcul with	$R_0 = 200, Q = 100, b = 20, K = 600, n = 7$

Identification on the test at $\dot{\varepsilon} = 10^{-2}s^{-1}$

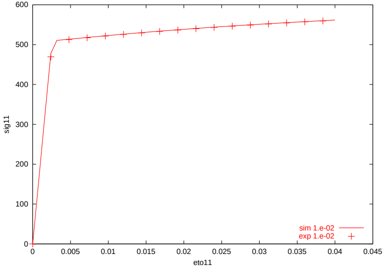


The following points are equivalent :

K	$R0$	Error
250	380	3.98e-06
325	340	2.06e-06
400	300	4.80e-06
425	290	2.97e-06
575	210	2.58e-06
600	200	5.55e-06
675	160	2.84e-06
750	120	6.28e-06

Reference	$R_0 = 250$, $Q = 100$, $b = 20$, $K = 500$, $n = 7$
Present calcul with	$R_0 = 120$, $Q = 100$, $b = 20$, $K = 750$, $n = 7$

Identification on the test at $\dot{\varepsilon} = 10^{-2} s^{-1}$

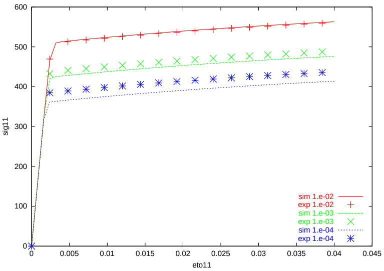


The following points are equivalent :

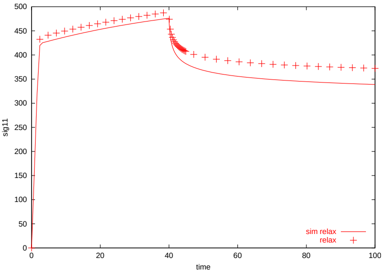
K	$R0$	Error
250	380	3.98e-06
325	340	2.06e-06
400	300	4.80e-06
425	290	2.97e-06
575	210	2.58e-06
600	200	5.55e-06
675	160	2.84e-06
750	120	6.28e-06

Reference	$R_0 = 250$, $Q = 100$, $b = 20$, $K = 500$, $n = 7$
Present calcul with	$R_0 = 380$, $Q = 100$, $b = 20$, $K = 250$, $n = 7$

Simulation of the whole data base



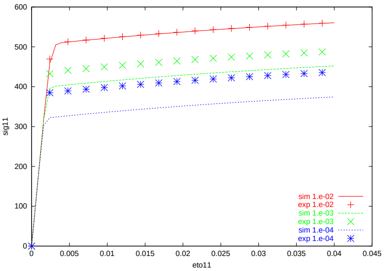
Hardening



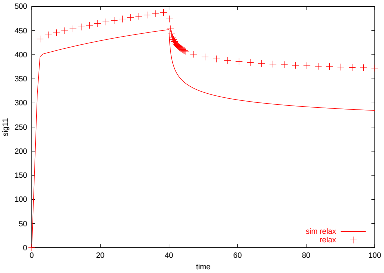
Relaxation

Reference	$R_0 = 250, Q = 100, b = 20, K = 500, n = 7$
Present calcul with	$R_0 = 200, Q = 100, b = 20, K = 600, n = 7$

Simulation of the whole data base



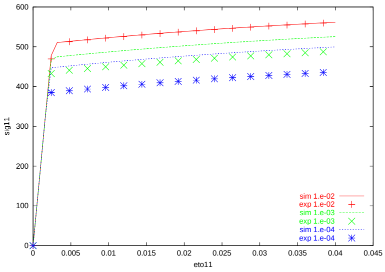
Hardening



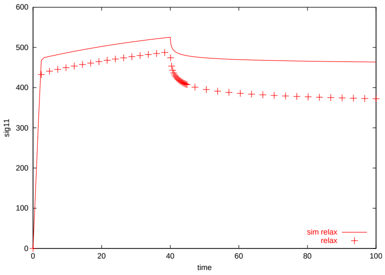
Relaxation

Reference	$R_0 = 250, Q = 100, b = 20, K = 500, n = 7$
Present calcul with	$R_0 = 120, Q = 100, b = 20, K = 750, n = 7$

Simulation of the whole data base



Hardening

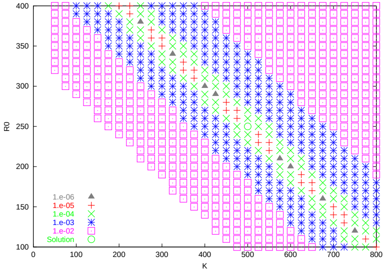


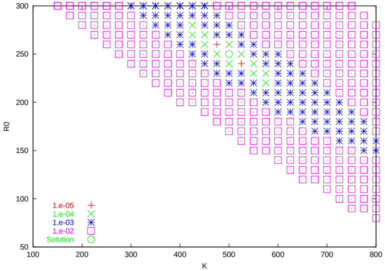
Relaxation

Reference $R_0 = 250, Q = 100, b = 20, K = 500, n = 7$

Present calcul avec $R_0 = 380, Q = 100, b = 20, K = 250, n = 7$

Comparaison of the cost functions



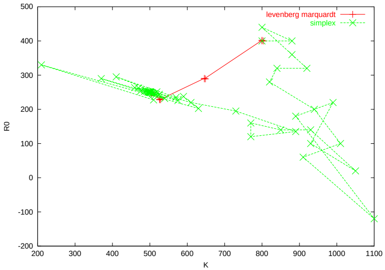


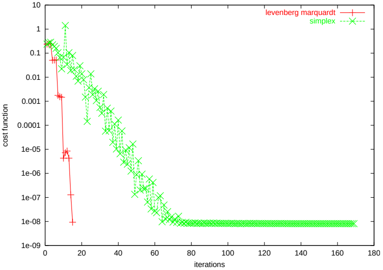
Test $\dot{\varepsilon} = 10^{-2} s{-}1$ only

Whole data base

Solution $R_0 = 250$, $Q = 100$, $b = 20$, $K = 500$, $n = 7$

Convergence of the algorithms





Convergence path History

Solution $R_0 = 250$, $Q = 100$, $b = 20$, $K = 500$, $n = 7$

Pratique de l'identification

- On ne peut espérer identifier correctement un coefficient que si ses variations ont un effet sur la réponse du modèle
- Pour une base expérimentale donnée, on peut obtenir des valeurs équivalentes de la fonctionnelle avec des valeurs différentes des paramètres
- Il faut prendre garde à ne pas "dévoyer" un modèle
  - …en faisant jouer à certains termes un rôle qu'ils ne devraient pas avoir, pour simuler un effet expérimental absent de la modélisation
  - …ou au contraire en introduisant par le modèle un effet dans la réponse numérique qui est probablement absent de la réponse expérimentale (on peut juger de la bonne tenue d'un modèle sans essai, mais avec du bon sens)
- Il faut libérer simultanément les paramètres couplés
- Il faut exploiter les possibilités de séparer l'identification en plusieurs étapes, et chercher à déterminer en priorité ceux qui ont un accès facile

Problèmes rencontrés lors d'un processus d'identification

- L'identification échoue (pour le minimum de la fonction–coût, l'accord simulation–expérience n'est pas bon)
  - *Vérifier l'expérience (!)*
  - La méthode d'identification n'est peut-être pas adaptée, *changer la méthode d'identification*
  - Le modèle n'est peut-être pas adapté, *changer le modèle*
- Non–unicité (trop de solutions)
  - La base expérimentale est trop pauvre, *ajouter des essais*
  - Le modèle a trop de degrés de liberté, *ajouter des contraintes*
- Utilisation du modèle hors de son domaine de validité (le domaine de travail des pièces en service n'est pas accessible par des expériences de laboratoire)
  - *Développer des approches plus physiques*
- Il n'est pas possible d'effectuer des essais simples sur élément de volume
  - *Utiliser des calculs de structures dans la boucle*